Discussion Paper No. 0107 March 2001

Multiple Juries and Two-Party Representative Democracy in the Condorcet Jury Framework

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Abstract

We consider decision regimes where two independent juries choose opposite majority rule winners in the same dichotomous choice problem. We highlight the role of the extent of victory on top of the numbers competence effects in evaluating the outcomes. We also analyze the judgmental competence of representative democracy in a two-party system when voter judgmental competence erodes with the size of the constituency. We show when constituency division enhances the polity's judgmental competence.

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I. INTRODUCTION

Marie Jean Antoine Nicolas Caritat, Marquis de Condorcet, was nothing if not supremely ambitious. He was one of the celebrated *encyclopaedistes* that intellectually prepared the way for the French Revolution. He subsequently became a president of the Legislative Assembly, all the while being a prominent member of the Academy of Sciences as a mathematician and philosopher. Eager to support his democratic proclivities with mathematical precision, Condorcet read a paper in the Academy on 17 July 1784 applying the Bernoulli theorem to majority voting (1785). Unlike other famous Academy colleagues and mathematicians Borda (of Borda voting) and Laplace (of Laplace Transform) who also investigated voting theory, Condorcet's exposition was convoluted and, thus, variously dismissed for so long. However, on the problem of dichotomous choice, his exertions were later recognized (Black, 1958) to form a solid positive (a.k.a., decision theoretic) foundation of democracy. The floodgates of research gradually opened up after Black (e.g., Grofman, Owen and Feld, 1981, 1989; Nitzan and Paroush, 1985; Ben-Yashar and Paroush, 2000).

In this paper, we will first present an exposition of the *Condorcet Jury Theorems* for the simplest regime of a single jury of homogeneous voters (Section II). In Section III, we will extend the regime to two independent juries and, in Section IV, we will compare the merits of direct democracy versus two-party representative democracy as to judgmental competence when constituency size erodes voter competence in representative elections.

II. THE CONDORCET JURY THEOREMS (CJT)

Since first rediscovered and used to great effect by Black (1958), the CJT has been generalized many times. Its current versions are highly involved and difficult (see, e.g., Ben-Yashar and Paroush, 2000; Owen, Grofman and Feld, 1989; Ladha, 1992). The version we present in this pedagogical section is the simplest and derives largely from Black.

1.Suppose society is faced with a dichotomous decision problem (A,B). "A" may be "Remove Erap" and "B" (not A) may be "Do not remove Erap". One is correct; the other, wrong but we do not know which one. We make the following Condorcet assumptions:

1. Homogeneity: society consists of n identical citizens; n is odd.
2. Independence: each citizen decides independently of every other citizen and votes only on the merits of the case.
3. Majority Rule: If A gets (n+1)/2 votes or better, A wins the election.

4. Judgmental Competence: each citizen has a probability v of making a correct choice, and a probability e of making the wrong choice with $v + e = 1$.

Let the number of citizens $n = h + k$, where h is the number of citizens deciding correctly and k the number of citizens deciding wrongly.

From the Bernoulli Theorem, the probability of h members making the correct decision is

$$\alpha = (h + k)!\,(h!k!)^{-1} v^h e^k.$$

The probability of the same h members deciding wrongly is

$$\beta = (h + k)!\,(h!k!)^{-1} v^k e^h.$$

The probability that the winning majority of h members ($h = (n+1)/2$ or better or $h > k$) is right is $\alpha(\alpha + \beta)^{-1}$ or (as in Black, op. cit.).

$$M = (v^{h-k})(v^{h-k} + e^{h-k})^{-1}. \qquad (1)$$

Since $n = h + k$, this can also be written as:

$$M = (v^{n-2k})(v^{n-2k} + e^{n-2k})^{-1}. \qquad (2)$$

M is the *judgmental competence* of the majority rule. It is the probability that the winner in a majority rule voting is the correct choice. We now state the *Condorcet Jury Theorems* for this set of circumstances (proofs are given not only for instructional purposes because these are not readily available but also because we will use these later as lemmas).

CJT (a):Vox Populi, Vox Dei:

Suppose $v > 0.5$ (better than even citizen competence), $h = (n + 1)/2$ or better and $k = k(n)$, $k' < 0.5$, $k'' \leq 0$: Then

$$\lim_{n \to \infty} M = 1.$$

That is, the probability that the majority vote will choose the correct option approaches certainty as n becomes very large.

Remark 1: k(n), k' < 0.5 means that the losing minority k can rise in number in a limited way as n rises. If k" = 0, k is a constant proportion of n.

Proof: Rewrite (2) as

$$M = [1 + (e/v)^{n\,(1-2k(n)n)}]^{-1}.$$

Note that $(1-2k(n)n^{-1}) > 0$ if $h = (n+1)/2$. But $k(n)n-1 \rightarrow \infty$ if $k' < 0.5$ and $k'' < 0$. Thus,

$$\lim_{n \rightarrow \infty} (e/v)^{n\,(1-2k(n)n^{-1})} = 0,$$

if $e < v$. But $e < v$ since $e + v = 1$ and $v > 0.5$. Thus, $\lim_{n\rightarrow\infty} M = 1$. Q.E.D.

This is also known as the *asymptotic CJT* for a competent homogeneous jury. If the competence v of each citizen strictly exceeds one half, the majority rule becomes infallible ("Vox Dei") as the citizenry size becomes infinite. One can immediately see how this legitimizes the pro-democracy belief of Marquis de Condorcet. But Marquis de Condorcet, who was of the Girondist party was arrested by the rival Jacobin rabble of the *Reign of Terror* and died abjectly in prison (before the guillotine could do its job!). He may have been a victim of the "diabolical" flip side of his own theorem:

CJT (b): Vox Populi, Vox Diaboli:

Suppose $v < 0.5$ (worse than even citizen competence), $h > k$ and $k = k(n)$, $k' < 0.5$, $k'' < 0$, then under our assumptions,

$$\lim_{n \rightarrow \infty} M = 0.$$

That is, the probability that the majority rule will choose the wrong decision approaches certainty as n becomes very large.

Proof: Note that

$$\lim_{n \rightarrow \infty} (e/v)^{n\,(1-2k(n)n^{-1})} = \infty$$

if $e > v$. But $e > v$ since $v < 0.5$ and $e + v = 1$. Thus,

$$\lim_{n \to \infty} M = 0. \qquad \text{Q.E.D.}$$

The role of citizen competence is extremely crucial. Majority rule, N large, is almost certain to choose the wrong option (thus, *Vox Diaboli*) if electorate's competence is lacking. The chaos and confusion that attended the aftermath of the French Revolution eventually paved the way to a dictator, Napoleon Bonaparte. In this atmosphere, a dictator of average competence v will do better than the electoral majority vote. Indeed, any citizen can be a dictator. This, incidentally, is the gist of the *non-asymptotic* CJT.

CJT (c): Vox Dictatoris, Vox Rationis:

Suppose $h > k$. then under our assumptions, $v > M$ iff $v < 0.5$.

<u>*Proof*</u>: Consider M in equation (1). Rewrite it as $M = [1 + (e/v)^{h-k}]^{-1}$. (i)

Sufficiency: Suppose $v > 0.5$ or $v > e$. $v > M$ can be written as:

$$[v + (e/r)^{h-k} v] > 1.$$

We only need to prove that $e < v(e/v)^{h-k}$. This resolves into

$$1 < (e/v)^{h - k - 1}. \tag{3}$$

Since $h = (n+1)2^{-1}$, $h-1 > k$. Thus, (3) is true, if $e > v$ and, in turn, this is true since $v < 0.5$. Thus, $v > M$. (ii) Necessity: Suppose $v > M$ with $h > k$. Then (3) is true and, thus, $v > e$ or $v > 0.5$. Q.E.D.

That Napoleon imposed a dictatorship over France was "right" (in the sense of better judgmental competence) if the French electorate then was less-than-even competent. The suggestion is not necessarily of a dictator but could be a democratic representative to decide on (A, B). Of course, the result that the republican Condorcet really wanted was the converse of the above, viz.,

CJT (d): VoxPopuli, Vox Rationis:

Suppose $h > k$. Then, under our assumptions, $v < M$ if and only if $v > 0.5$.

<u>*Proof*</u>: Same as above with the inequalities reversed.

CJT(a) and CJT(d) are viewed by many (Urken, 1991; Hewill and McLean, 1994) as the normative foundations of democracy. Even with finite size citizenry, majority vote's judgmental competence will beat the judgment of a dictator or of any single citizen with average competence. More heads are better than one! Of course, if the dictator's competence is above average, one will need the numbers effect to realize the superiority of majority vote.

In fact, suppose the dictator's competence is ϵv, $\epsilon > 1$. One can always find the size of the jury, n^*, so that at $n \geq n^*$, majority rule competence exceeds ϵv of the dictator. Note that since the citizenry is homogeneous as to judgmental competence, the dictator must be an import from another polity, a fact not uncommon in history (e.g., William of Orange who became King of England; the brothers Napoleon who were installed as kings of Spain, Holland and Northern Italy). A propos this question, the following are of interest:

Corollary 1: Suppose the imported dictator's judgmental competence is ϵv, $\epsilon \geq 1$, $\epsilon v < 1$, $k = \delta n$ and $v > 0.5$ is the citizen competence. Then the majority rule's competence exceeds the dictator's if $n > n^* > 0$, where

$$n^* = [\log (1 - v\epsilon) / v\epsilon] [(1-2\delta) \log (e/v)]^{-1}. \quad (4)$$

Proof: Set $\epsilon v = M = [1 + (\epsilon/v)^{n(1-2\delta)}]^{-1}$. Solving for $n = n^*$ gives (4). By CJT(a), n^* always exists since $\epsilon v < 1$, Q.E.D.

Note that if $\epsilon > 1$, $(1 - v\epsilon)(v\epsilon)^{-1} < 1$ and $(e/v) < 1$ if $v > 0.5$. Thus, $n^* > 0$. Of course, if $v < 0.5$, and $\epsilon v > 0.5$, number will not help majority rule. We have the following:

Corollary 2: A large *democratic rabble deserves a dumb king*, i.e., if $v < 0.5$, an imported dictator with judgmental competence $v > \epsilon v > 0$, $\epsilon < 1$, is better than a boundlessly large democratic rabble.

Proof: Obvious from CJT(b).

If the electors decide only by tossing a fair coin, there is no numbers effect.

Corollary 3: *Safety in numbers* is no refuge *for the average electorate*, i.e., $M = 0.5$ for all n if $v = 0.5$.

The following property is well-known (e.g., Black, 1963; Owen, Grofman and Feld, 1983) but we state it for completeness:

Property 1: Majority rule competence M rises as v rises.

Proof: From the definition of M in (1), we get $M'(v)$ noting that $e = 1 - v$. This gives $X^2 (h - k) (v + e)^{h-k-1} v^{-(h-k+1)} > 0$. Q.E.D.

Black's proof has "$v^{(h-k-1)}$". The sign is, however, preserved. Thus, a rise in citizen competence always raises the majority rule competence M, regardless of the initial level of v. Education and information access serve the democratic ideal based on majority rule. In contrast, ignorance and information repression legitimize the claim of a dictator and, thus, are often used. The following is also well-known and is in Black (op.cit.) but the correct proof is given here:

> _Property 2_: The closer to unanimity is the vote (i) among competent ($v > 0.5$) citizens, the higher is majority rule's judgmental competence, i.e., M rises as h rises, n being constant; (ii) among incompetent ($v < 0.5$) citizens, the lower is M.

Proof: (i) Note that $k = n - h$. Substituting in equation (1) gives:

$$M(h) = (v^{2h-n})(v^{2h-n} + e^{2h-n})^{-1} = [1 + (e/v^{2h-n}]^{-1}.$$

Let $y(h) = (e/v)^{2h-n}$ so that $M'(Y) = -(1+y)^2 < 0$ and $y'(h) = 2y \log (e/v) < 0$ if $v > 0.5$. Substituting, we get:

$$M'(y'(h)) = 2M^2 (e/v)^{h-k} \log (v/e) > 0.$$

(ii) The sign reverses if $\log(v/e) < 0$. Q.E.D.

Black's proof lacks the "squared" in (e/v^2) but the sign is again preserved.

The CJT has been progressively generalized in many directions. In particular, (i) if citizens are heterogeneous and $\bar{v}$ is the *mean competence* of the voters, the asymptotic CJT's (a ,b) all hold with $\bar{v}$ substituted for v, however skewed the distribution around $\bar{v}$ (Owen, Grofman and Feld, 1981; 1989); (ii) if $\bar{v}$ is the competence of any member randomly selected from a set of heterogeneous voters, majority rule judgment $M > \bar{v}$ (Ben-Yashar and Paroush, 2000). We will use these results in the subsequent sections. Ladha (1992) allows for some correlatedness in voting.

III. TWO DISTINCT JURIES

So far, we have only either restated or elaborated on known results related to the judgmental competence of majority rule in a homogeneous single jury. We now push the envelope farther. There are contexts where a complete outsider O has to choose between A and B. By *complete outsider* here we mean a total lack of information on the merits of A or B. But O may have certain second order information, i.e., information about other people or juries who do have first-hand information. Let there be two distinct sets of people or citizens (juries). Let us call the first one P_1 and the second P_2. O knows the following about P_1 and P_2:

(i) P_i consists of N_i homogeneous members of judgmental competence v_i, i = 1, 2.
(ii) Each jury chooses between A and B by majority vote following assumptions in Section II.
(iii) O observes the outcomes of these votes: (1) as to which wins, and (2) by what majority.
(iv) Each jury decides independently of the other.

O's goal is to maximize the probability of a correct decision on his part given the second order facts at his disposal.

There are many real life situations where "the outsider" parable is relevant: (i) the ordinary citizen vis-à-vis the two juries called the *lower house* and the *upper house* of the legislature on an esoteric subject of, say, power industry restructuring; (ii) a patient vis-à-vis two sets of doctors -- one from Mayo Clinic, another from a PGH; the uncommitted citizens vis-à-vis the "pro-Resign" and the "anti-Resign" advocates. We now have the following:

Claim 1: Suppose $v_1 > v_2 > 0.5$ and $N_1 < N_2$. O should choose the winner of the P_1 election if and only if

$$N_1(1-2\delta)\,[\log(e_1/v_1)] \geq (N2(1-2\delta_2)\,[\log(e_2/v_2)] \qquad (5)$$

where $\delta_i = (k_i/N_i)$ and k_i is the losing minority in i = 1, 2. O chooses the winner of P_2, otherwise.

Proof: If A wins in both P_1 and P_2, then O maximizes probability of correct decision by choosing A. If, however, A wins in P_1 and B in P_2, a conflict arises. The judgmental competence of majority vote in P_1 is $M_1 = [1 + (e_1/v_1)\exp[N_1(1 + 2\delta_1]^{-1}$ while in P_2 is $M_2 = [1 + (e_2/v_2)\exp[N_2(1-2\delta_2)]^{-1}$ where $k_i = \delta_i N_i$, i = 1, 2, and $0 \leq \delta_i < 0.5$. $M_1 > M_2$ if

$$(e_1/v_1)\exp[N_1(1-2\delta_1)] < (e_2/v_2)\exp[N_2(1-2\delta_2)]\,.$$

Since $v_i > 0.5$, $e_i < v_i$, $i = 1, 2$. Taking the log of both sides reverses the sign. Thus

$$N_1(1\text{-}2\delta_1) \log(e_1/v_1) \geq N_2(1\text{-}2\delta_2) \log(e_2/v_2).$$

The probability of getting it right is maximized if O chooses the winner in P_1 since $M_1 > M_2$ (only if) Suppose the latter. Then inequality (5) holds. Q.E.D.

O's probability of choosing right is to take the winner in the more competent, if smaller, group P_1 if the inequality holds. The inequality holds even if $N_i = N_2$ and $v_1 = v_2$ if $\delta_1 < \delta_2$, i.e., the majority in P_1 exceeds the majority in P_2. Thus, we have a special case of Claim 1:

Corollary 4: (i) Suppose $v_1 = v_2 > 0.5$ and $N_1 = N_2$, then O should choose the winner in P_1 if and only if $\delta_1 < \delta_2$. (ii) Suppose $v_1 = v_2 < 0.5$, $\delta_1 = \delta_2$. O should choose the loser in P_2 if $N_2 > N_1$.

Proof: (i) (if) The inequality (.) becomes $(1\text{-}2\delta_1) > (1\text{-}2\delta_2)$ which is true if $\delta_1 < \delta_2$ (only if) is obvious. (ii) If $N_2 > N_1$, and $v_1 = v_2 < 0.5$, $M_2 > M_1 < 0.5$. Thus $(1\text{-} M_2) > (1\text{-} M_1)$, the probability of the loser being correct is higher in P_2. Q.E.D.

The question as to which jury to believe is a lively one. In our case, P_1 may consist of a much smaller population but with a slightly better competence. The size of the majority in each case is crucial. If P_1's citizens are more solid behind its winner than P_2's, then correctness is more likely with the winner in P_1.

Claim 2: (i) If $v_1 > 0.5$ and $v_2 \leq 0.5$ (i.e., P_2 is completely ignorant or worse), O should choose the winner of P_1. (ii) If $0.5 > v_i > v_2$, and $N_1 \leq N_2$, and $\delta_1 \leq \delta_2$, O should choose the loser of P_2.

Proof: (i) From CJT(c), $v_2 > M_2$ and $M_1 > v_1$. So $M_1 > M_2$ and the winner of P_1 has the greater probability of being right. (ii) If $0.5 > v_1 > v_2$ and $N_1 = N_2$, $\delta_1 = \delta_2$. Thus, the loser in P_2 has a greater probability of being right $(1 - M_2)$ than the loser in P_1 with $(1\text{-}M_1)$. Raising N_2 lowers M_2; raising δ_2 lowers M_2 by Property 2. Thus, O chooses the loser in P_2. Q.E.D.

Where P_1 and P_2 are both incompetent, the loser in the less competent population has a better chance of correctness.

IV. DIRECT VERSUS TWO-PARTY REPRESENTATIVE DEMOCRACY

Heterogeneity in judgmental competence in the population comes in two forms: (1) over a particular dichotomous issue (A, B), judgmental competence can vary across voters, and (2) across various dichotomous issues (A_1, B_1), (A_2, B_2), etc., the judgmental competence of one voter can vary. In the latter, one voter can be better informed about one issue than about another. These two types of heterogeneity underpin the institution called *representative democracy* -- a regime where a majority vote of the electorate determines a *representative* invested with the power to decide over (A, B). This contrasts with *direct democracy*, a regime where the electorate decides on (A, B) directly by a majority vote.

We adopt the following notations and assumptions:

(a) The electorate is of size N and heterogeneous (unless explicitly replaced) as to judgmental competence. The distribution of competence on (A, B) has a finite mean v_d and is symmetric around the mean.

(b) The *average* judgmental competence of the electorate on (A, B) is v_d and the judgmental competence of the direct majority vote is M_d.

(c) The electorate may alternatively choose between two members T and Q standing for election by majority vote as a *representative* with power to decide on (A, B). We refer to the election contest between T and Q as (T, Q). (We ignore the mechanism for choosing the two candidates from the population. A two party system may be in place).

(d) The judgmental competence of T and Q are t and q, respectively.

(e) Let the judgmental competence of the electorate over (T, Q) be distributed symmetrically with finite mean v_r. The judgmental competence of majority vote on (T, Q) is M_T.

(f) Let M_r be the judgmental competence of deciding over (A, B) via the elected representative.

(g) We assume that electorates and the representative decide independently of others and only on the merits of the case.

The comparators here are M_d and M_r, the judgmental competence over (A, B). Note that $M_r = tM_T + q\,(1 - M_T)$, that is, M_r is the weighted average of the two candidates' competence over (A, B) weighted by the majority vote competence over the two candidates.

A. Representative Versus Direct Democracy

Claim 3: Suppose $v_d < 0.5$ on (A, B) and $v_r > 0.5$ on (T, Q). If $t > q > 0$, then $M_r > M_d$ as $N \to \infty$.

Proof: Let M_T be the probability that T gets elected by a majority vote. Then, $(1 - M_T)$ is the probability that Q wins. Here, T is the correct choice since $t > q$. The judgmental competence of representative democracy is

$$M_r = rM_T + q(1-M_T).$$

If $v_r > 0.5$, $M_T \to 1$ as $N \to \infty$ by CJT(a) and $M_r \to t > 0$. If $v_r < 0.5$, $M_T \to 0$ as $N \to \infty$ and $M_r \to q$. But by CJT(b), direct democracy (voting) has judgmental competence $M_d \to 0$ as $N \to \infty$ since $v_d < 0.5$. Q.E.D.

Thus, for very large groups of heterogeneous competence, electing a representative among themselves to make the decision over (A, B) for which average competence of the electorate is low is better than a direct voting on (A, B).

Claim 4: Let every voter be a fair coin-toss decider over (A, B), i.e., $v_d = 0.5$. Let $t > 0.5$ and $q < 0.5$. Then as $N \to \infty$,

(i) $M_r > M_d$ if $v_r > 0.5$

(ii) $M_r < M_d$ if $v_r < 0.5$.

Proof: If $v_d = 0.5$, $M_d = 0.5$. If $v_r > 0.5$, $M_r \to t > 0.5$ as $M_T \to 1$. Thus, $M_r > M_d$, which is (i). The second follows since $M_T \to 0$ and $M_r \to q < 0.5 = M_d$. Q.E.D.

In the case where voters truly recognize their own ignorance and would simply toss a fair coin, one or both of the candidates had better be better-than-average and the voters' discernment of this fact had better be better than even for representative democracy to improve on direct voting.

Claim 5: Suppose $v_r > 0.5$ and distribution is symmetric around 0.5. Suppose $v_d > 0.5$ on (A, B) and the distribution of judgmental competence is symmetric around 0.5. Then $M_r > M_d$ if $t + q > 1$.

Proof: Since $v_T > 0.5$, and $M_T > v_T$ by the non-asymptotic CJTof Ben-Yashar and Paroush (2000). Suppose $t + q > 1$ or $q > 1\text{-}t = (0.5 - 0.5t)(0.5)^{-1}$. Since $t > 0.5$, this inequality still holds if we substitute M_T for 0.5. Thus, $q > (0.5 - tM_T)(1\text{-}M_T)\text{-}1$ and $q(1\text{-}MT) + tM_T > 0.5$. Thus, $M_T > 0.5 = M_d$ since average competence on (A, B) is $v_d = 0.5$ and the distribution around 0.5 is symmetric. Q.E.D.

Representative beats direct democracy if the combined competencies of the two candidates for representative exceed one where the averages $v_r > 0.5$ and $v_d = 0.5$ and distribution is symmetric around the mean and one candidate is competent.

Claim 6: For a given judgmental competence t of candidate T, the greater is v_T on (T, Q), the smaller is the competence of Q required to preserve M_r.

Proof: Let $M_r = M_r^o$, a constant. Solving for q we get $q = (M_r^o - tM_T)(1\text{-}M_T)^{-1}$ and $(\partial q/\partial M_T) = [(1 - t)M_T - 1][1\text{-}M_T]^{-2} < 0$. But $\partial M_T/\partial v_T > 0$ from (Property 1). Thus $(\partial q/\partial v_T) < 0$ for unchanged M_r.

Thus, the more competent is the electorate as to the qualifications of the candidates, given the competence of the first candidate, the more dumb can the second be without eroding the judgmental competence of representative democracy.

Claim 7: Suppose $v_r = v_r(N)$, $v_r' < 0$, and $v_r(N^o) < 0.5$, $N^o < N$. Suppose $v_d = 0.5$ and the distribution of competence around 0.5 is symmetric. Suppose $t > 0.5 > q > 0$. Then, $M_r < M_d$ as $N \rightarrow \infty$.

Proof: $M_T \rightarrow 0$ as $N \rightarrow \infty$ and $M_r \rightarrow q < 0.5$. But $M_d = 0.5$. Q.E.D.

If average competence over (T, Q) erodes with electorate size so that as N becomes very large, the average falls below even, then direct voting by coin-tossing electorate improves on representative voting under majority rule.

B. Districting: Homogeneous Voters

Smaller democracies or voting units are better than large ones if information on candidates erodes with number. We now inquire why *districting* or division of the constituency into smaller voting units serves competence.

Claim 8: Suppose N is homogeneous but finite. Suppose v_r decreases with the number of voters, i.e., $v_r(N)$, $v_r' < 0$, and for small enough $N^o < N$, $v_r(N^o) > 0.5$ but $v_r(N) < 0.5$. Let $N_w = (N/w) \leq N^o$, with $w \geq 3$ and odd. Each w is called a *district*. Suppose that at every district w, voters choose a representative by majority vote from (T, Q) with judgmental competence t and q, $t + q > 1$, $t > q$. Then, *representative democracy with districting* with judgmental competence M_{rr} beats representative democracy without districting.

Proof: Since $v_r(N_w) > 0.5$, $M_{Tw} > 0.5$ and $M_{rw} > 0.5$, this is true for every district $w \geq 3$ and odd. So there are w representatives each with $M_{rw} > 0.5$. The overall judgmental competence on (A, B) of the w representatives deciding by majority rule is $M_w > 0.5$. Since $v_r(N) < 0.5$, $M_{TN} < v_r(N) < 0.5$.

$$M_{rN} = tM_{TN} + q(1 - M_{TN}) < tM_{rw} + q(1 - M_{Tw}) = M_{rw},$$

since $t > q$ and $M_{TN} < M_{Tw}$. Q.E.D.

When competence as to candidate qualification erodes with constituency size, it is beneficial to subdivide a large voting constituency into smaller ones.

V. COSTLY INFORMATION and the INFORMATION DIVIDE

The results so far are hardly interesting if information is everywhere free and the capacity or aptitude to process information is acquirable at zero cost. In reality, however, valuable information is costly and the capacity to process information is even more so. This is true even in dichotomous choice issues and referenda. The population may, therefore, be segmented by an *information divide*. If such a divide can be objectively identified, equal weighing of votes does not make sense for optimal judgmental competence (Nitzan and Paroush, 1985). This is especially true in *medicine*. The patient becomes almost the *Outsider* of Section III and the doctor's vote gets all the weights. Sometimes, a second or third opinion is sought but always to the exclusion of non-expert. This is true of all *expert advice* situations, whether legal accounting, architectural, etc. Similarly, stock ownership determines the number of stockholder votes presumably recognizing the incentives compatibility of information investment and votes.

The problem with the information divide is its *non-observability*. Who belongs to which side of the divide is subject to *asymmetric information*. Citizens know where they themselves belong (i.e., whether they are toss-coin variety) but won't know where others belong. Neither does the state. Thus, on most referenda, the state interprets the will of the people as the majority rule with one-man-one-vote, recognizing its inability to discriminate. "All men are created equal," was not spoken from omniscience but from ignorance. This could easily be subject to CJT (b).

To remedy this, *representative democracy* is partly the answer. The general electorate from a geographic unit votes for a *representative*. This representative, in theory, should satisfy only two criteria: a large capacity for information processing to ensure $v > 0.5$, and the capacity to decide independently (one of the CJT assumptions). The electorate's competence over candidates for representative is presumably better than over more complex issues. Given these, representative democracy improves the decisional competence of the polity. The representatives operate as an *expert* group tasked as agents to decide on issues get all the weights by the principal, the general electorate, which only conforms. When voter competence over candidates improves the smaller is the voting constituency, *districting* with multiple representatives is *better than a single representative.* This is predicated on the Condorcet assumption that decisions are made only on the merits of the issues.

This is also where representative democracy falters. The information set over candidates can be murkied by irrelevant information or interest. The electorate may vote to office representatives with zero capacity to process information so that $t < 0.5$ on issues. The set of candidates may be very poor (t, $q < 0.5$). This could be because voters either do not or cannot properly discriminate among candidates (their own $v < 0.5$) or they sell their votes to the highest bidder. Furthermore, representatives may, even if competent, in turn, sell their own votes on issues or pursue selfish interests, thus, confounding CJT(d). In this state of affairs, majority rule democracy triggers a cascade of consistently inconsistent decisions, which undermine the long-term common good, say. The system is soon headed for a collapse, which invites the emergence of either a new social contract, or a dictator.

Thus, *competence of the citizenry*, *quality information* and a *modicum of morality* are prerequisites for a representative democracy premised on majority rule. Because these are costly, economic poverty is a poor setting for electoral democracy. This does not mean that a dictator will always do better. But this is a good place for a dictator to prospect. And clearly, it is to his interest to perpetuate it.

VI. CONCLUSION

We extend the Condorcet Jury framework to dichotomous choice problems with two juries and to regimes where the electorate decides indirectly on the issue by choosing a representative or representatives who vote directly on the issue. We specialize on the case of the two-party representative democracy, especially where the competence of the electorate vis-à-vis the candidates erode with the size of the constituency.

We show in the case of two juries how the extent of victory in each case can nullify or reinforce the effect of jury size and differential competence on majority votes' judgmental competence. We show when representative democracy beats direct democracy. Finally, we show how *districting* improves the judgmental competence in representative democracy.

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