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ON PREDICTING SUBSECTORAL OUTPUTS AND PRICES VIA AN OPTIMIZING MODEL

by

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The possibility of using an optimizing (maximum) problem to solve explicitly for competitive equilibrium prices and outputs was apparently first suggested by Samuelson (1952). Subsequently, Duloy and Norton (1973) applied this idea to Mexican agriculture. In this note we wish to look into the merits of employing a similar device as a model to predict market prices and outputs in the Philippine agricultural sector.

For simplicity, suppose only two subsectors (say rice and non-rice) and three inputs (say land, labor and capital). Generalization of the argument to more than two outputs and more than three inputs will be apparent.

Let

(1) $p_i = g_i(y_i) \qquad i = 1, 2$

(2) $y_i^o = f_i(x_{1i}, x_{2i}, x_{3i}) \qquad i = 1, 2$

(3) $y_i = y_i^o \qquad i = 1, 2$

(4) $x_j = x_{j1} + x_{j2} \qquad j = 1, 2, 3$

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*The writing of this note was occasioned by discussions with David E. Kunkel and the model formulated here is basically similar to one that Kunkel describes in a paper he presented at the Third Agricultural Policy Conference held in U.P. Los Baños in October 1975.

Inverse demand functions are given in (1), the demand functions being expressed by $y_i = g_i^{-1}(p_i)$. Production functions are given in (2), x_{ji} being the amount of input j used in i production. Equilibrium obtains with (3) when amount demanded equals output. In (4), x_j is the total amount of input j supplied and used.

Assume that a fixed amount of input 1 (say land) is available for allocation between the two subsectors. Input 2 (say labor in a surplus labor market) is available in any amount at a given fixed price π_2 (which may be institutionally determined, in the case of labor, or determined in a world market, as in the possible case of fertilizer). Input 3 (possibly capital) is available to the sector at an increasing supply price, $x_3 = h_3^{-1}(\pi_3)$, or

(5) $$\pi_3 = h_3(x_3)$$

These three cases--a fixed amount of a resource, a fixed price, and an increasing supply price--cover the possibilities.

Consider now the problem of maximizing the objective function

(6) $$\phi = \sum_{i=1}^{2} [\int_0^{y_i'} p_i dy_i + \lambda_i(y_i^o - f_i(x_{1i}, x_{2i}, x_{3i}))$$

$$+ \mu_i(y_i - y_i^o)] - \pi_1(x_{11} + x_{12} - x_1) - \pi_2(x_{21} + x_{22})$$

$$- \int_0^{x_3'} \pi_3 dx_3$$

where λ_i, μ_i, π_1 are Lagrange multipliers. ϕ is basically the difference between two sums: the sum of areas under demand curves for outputs, and the sum of areas under supply curves for inputs (with the exception of input 1 whose amount is assumed fixed). We note that the term in ϕ involving π_2 could also be written as

$$-\int_0^{x_2'} \pi_2 dx_2$$

Assuming an interior solution, necessary conditions include

$$(7) \quad \partial\phi/\partial y_i = p_i + \mu_i = 0 \qquad i = 1, 2$$

$$(8) \quad \partial\phi/\partial x_{ji} = -\lambda_i \partial f_i/\partial x_{ji} - \pi_j = 0 \qquad i = 1, 2; \; j = 1, 2, 3$$

$$(9) \quad \partial\phi/\partial y_i^o = \lambda_i - \mu_i = 0 \qquad i = 1, 2$$

from which follow

$$(10) \quad p_1 \partial f_1/\partial x_{j1} = p_2 \partial f_2/\partial x_{j2} = \pi_j \qquad j = 1, 2, 3$$

which are the familiar conditions equating marginal value products to input prices. A solution of the model thus implies (10), which indeed is the rationale for the objective function (6).

Suppose, then, that we have correctly specified and empirically estimated functions for (1), (2) and (5), as well as values for x_1 and π_2. Then a programming algorithm that yields maximum ϕ also gives predicted values for the y_i and p_i. Abstracting from statistical and estimation errors, how well would such a model predict? Essentially, for the model at hand, this question is the same as: How closely does the assumption of a competitive equilibrium approximate the real world (of the Philippine agricultural sector)?

We know that in a competitive equilibrium, shadow prices generated by an (appropriate) optimizing model would equal equilibrium (market) prices. To the extent, therefore, that our real world departs (which it does) from the assumptions characterizing a competitive equilibrium, we can expect _a priori_ a divergence of shadow prices from market prices (to which actual markets respond). Accordingly, with marginal value products being equated to the shadow price π_1 in (10), the model's solution would give us wrong predictions of actual prices and outputs. To what degree they are wrong cannot of course be determined _a priori_. Only comparisons of the model's predictions and actual market values could possibly provide information on this point. At the least, however, we can say that the model's predictions will in general be different from actual values--even after abstracting from specification, estimation and statistical errors. The reason, simply, is that actual markets do not satisfy all the assumptions of a competitive equilibrium, which in this model takes on a very heavy burden in addition to its role as a theoretical framework useful for making _qualitative_ predictions in empirical applications.

The point is this: In empirical work we do make predictions of how some economic variables would change in response to changes in some parameters, and we (often) use competitive equilibrium assumptions (of the comparative statics kind) in making those predictions as to the directions of change. We do not make _quantitative_ predictions unless we have some econometric model, or good intuition which is sometimes better, as basis for such predictions. The econometric models that we use for this purpose are estimated from actual data that reflect both competitive and non-competitive equilibrium and disequilibrium aspects of the real world, and these models do not make essential use of competitive equilibrium assumptions. But this is not the case with the model at hand, which requires equations (10) for making quantitative predictions, and equations (10) presuppose all the competitive equilibrium optimizing properties for shadow prices to equal market prices.

We conclude that an optimizing model based on competitive equilibrium properties is not suitable for quantitative prediction purposes. But note, however, that this conclusion is based on the observation that shadow prices will in general differ from market prices. So if we can eliminate shadow prices in the model's solution, the situation could possibly be improved. One possible way out of this difficulty is to estimate independently the x_{1i} (for our fixed input 1) and deleting the term involving π_1 in (6), making the x_{1i} given values in the optimization problem. Then there would be no shadow price π_1 that the solution would require to equal input 1's market price.

References

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