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# A NOTE ON SENSITIVITY PROPERTIES OF FOUR INCOME INEQUALITY MEASURES

by

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It is generally agreed upon that no single summary measure of income inequality is adequate. [1, 2, 8, 9] Accordingly, several measures should be employed. Our purpose, in this note, is to comment on certain properties of some inequality measures.

From the variety of measures at our disposal, we select four: 1) the Gini Concentration Ratio (CR); 2) the Index of Decile Inequality (IDI) or more generally the Oshima Index (OI); 3) the coefficient of variation (CV); and 4) the standard deviation of the logs of income (SL). This combination, we feel, can be used in a complementary fashion in making intertemporal or interspatial comparisons of the distribution of income.

The CR is defined as the relative mean difference which is a weighted average of differences between values of all possible pairs of observations in a series. The coefficient can also be defined geometrically by means of the familiar Lorenz curve. [5, 9] Although the CR is the most widely used measure of inequality, several difficulties are found in its application. Ambiguities arise when two Lorenz curves intersect, that is, one can find the same value of inequality for two very different curves. Another common complaint lies in the insensitivity of the measure. [2, 6, 7, 8]

The OI which was suggested as an alternative to the CR by Oshima

[7] is analogous to the mean deviation and is divided by a scale factor to set an upper bound of 1.0. If the size distribution is arrayed in decile shares, the coefficient is calculated by simply taking the sum of the deviations of each decile share from 10% and dividing by 180, which yields the IDI. It has been argued that the OI is insensitive to changes in the distribution that occur other than in the middle income range. [4] Conversely, Mangahas has argued that the OI is biased toward the moderate income range since the measure is more responsive to changes that occur within this region rather than in the extreme regions. [3] We extend the argument and demonstrate below that the OI is the most sensitive of the four measures to changes in the distribution affecting the middle income brackets.

The coefficient of variation is another commonly used measure of income inequality. It is formed as the ratio of the standard deviation of incomes to the arithmetic mean of incomes and, hence, is independent of the units employed. Taking the second moment about the mean accords larger deviations greater weight and since income distributions are highly skewed right, the measure attaches greater importance to incomes found in the upper brackets. Consequently, changes that occur within this region should be best reflected by the CV. [1, 6, 7]

Our fourth dispersion coefficient is the standard deviation of the logs of incomes. By introducing logarithms, absolute differences or changes are transformed into percentage differences or changes and because of skewness (before transformation), absolute changes of low

incomes will have relatively greater influence on this measure than will absolute changes of high incomes. Stated differently, the SL should show greater sensitivity to changes in income that occur in the lower income brackets. [1, 2, 6, 7]

The literature has commented on all of the above dispersion coefficients, however, the arguments have usually been on biases and on the inadequacies of certain measures and has oftentimes resulted in a search for the "best" overall measure of inequality. We take the view that the measures have properties such that one measure is not preferred to the others at all times but that each measure will aid in attempting to explain the distribution of income by size and changes in the distribution. The biases of the measures we prefer to call "sensitivity" properties and we demonstrate these properties through a hypothetical exercise.

For this exercise, we first construct an income distribution (a frequency distribution of households and the corresponding income accruing to each income bracket). We increase Personal Income (by 3%) and then specify to which income groups this increase will go to. We calculate the four measures after each simulation. Our first construction is based on actual data for Japan in 1956, although we have beforehand grouped the families into deciles and have rounded average income per income class to facilitate calculations.

To get an overall view of these sensitivity properties, we first stipulate that the highest quintile (deciles 9 and 10) receives equally the 3% increase in Personal Income, then the second highest quintile, the

third, the fourth, and finally the lowest quintile. We should find that our inequality measures decline as we proceed through each simulation. On Chart 1 we have indexed all of the measures at the point of greatest inequality (the increase going to the highest quintile). As can be seen, all measures show a narrowing of inequality. Our first observation is that the CR declines in straight line fashion, that is, it shows equal sensitivity through each simulation. [While we have said that no one measure is preferred to the others at all times, perhaps it is justifiable to say that for all times the CR is preferred.] The IDI is indeed insensitive at the extreme ranges (first through first deciles at the low end and seventh through eighth at the upper end) but is the most sensitive between simulations involving the middle income brackets. The CV exhibits the greatest changes at the upper end of the distribution but is the least sensitive to changes occurring below the seventh decile. The SL, on the other hand is the least sensitive to simulations involving the highest three quintiles but is, by far, the most sensitive to changes involving the lowest income groups.

To further illustrate our point, we specify differing degrees of inequality or equality in order to better understand these properties. On Chart 2 we index the measures with the original distribution and simulate three degress of inequality (or equality in reverse order). Moderate inequality means that we have given the 3% increase in Personal Income to the highest three deciles equally. For Greater inequality we stipulated that the highest decile receive 50% of the increase, the ninth decile 40%, and the eighth only 10%. In Greatest inequality the highest

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in the degree of inequality. The CR and the SL show a
the changes are negligible. The IDI is completely inse
changes in assumptions.

On Chart 3 we stipulate changes at the other end
spectrum. Moderate equality means that we have given
to the three lowest deciles equally. For Greater equal
given the lowest decile 50% of the increase, the second
third 10%. For Greatest equality the lowest decile rece
gain and the second 40%. (We could not give the lowest
gain for they would no longer be the lowest.) As expec
indicates changes in our assumptions involving the deg
(equality) at the lower end. Again the CR shows a decl
through each simulation but it is not as sensitive as the
negligible change while the IDI is invariant.

Changes in the degree of inequality involving the
groups is illustrated on Chart 4. We again label our fi
Moderate equality where the fifth decile receives 10% o
sixth receives 40%, and the seventh 50%. For Greater
three deciles share equally the gain, and for Greatest
Moderate equality allocation is reversed. All measure
however, the IDI shows the greatest sensitivity in this

When the economy is growing the actual distribut
income is a complex process where all strata in the s

absolute terms and it is not easy to discern which groups gain relative to the others. It is conceivable that our measures show inconsistencies. For instance, if the lowest and the highest income groups lose and the middle income groups gain, the SL will show greater inequality (because of the loss of the lowest groups), the CV will indicate greater equality (because of the loss of the highest groups), the CR may remain constant (if the losses cancel), while the IDI would fall or increase depending on how the gains in the middle groups were allocated. Can we say that inequality has widened? or narrowed? The use of a single measure is clearly inadequate. The use of all four measures, while not answering the question, certainly provides more information as to the changing shape of the curve. It will be society's task to answer the question.

[1] Kravis, Irving. The Structure of Income, Some Quantitative Essays, University Press, 1957.

[2] Kuznets, Simon. "Quantitative Aspects of the Economic Growth of Nations: VIII. Distribution of Income by Size," Economic Development and Cultural Change, XI, 2, Part II (January 1963).

[3] Mangahas, Mahar. "A Note on "Income Inequality and Economic Growth: The Postwar Experience of Asian Countries"," Institute of Economic Development and Research Discussion Paper No. 72-27, (February 1973).

[4] McCleary, William A. "On Measuring the Degree of Inequality in the Distribution of Income: Comment of Professor Oshima's Article," paper presented at a Workshop on Income Distribution in Manila sponsored by the ADC and CAMS, January 1972.

[5] Miller, Herman P. Income Distribution in the United States. Washington D.C.: U.S. Government Printing Office, 1966.

[6] Oshima, Harry T. "International Comparisons of Size Distribution of Income with Special Reference to Asia," Review of Economics and Statistics, XLIV, 4 (November 1962).

[7] _____. "Income Inequality and Economic Growth: The Postwar Experience of Asian Countries," Malayan Economic Review, Vol. XV, 2 (October 1970).

[8] Weisskoff, Richard. "Income Distribution and Economic Growth in Puerto Rico, Argentian, and Mexico," Review of Income and Wealth, Series 16, Number 4, December 1970.

[9] Yntema, Dwight. "Measures of the Inequality of the Personal Distribution of Wealth or Income," Journal of the American Statistical Association, XXVIII, 184 (December 1933).

Chart 1

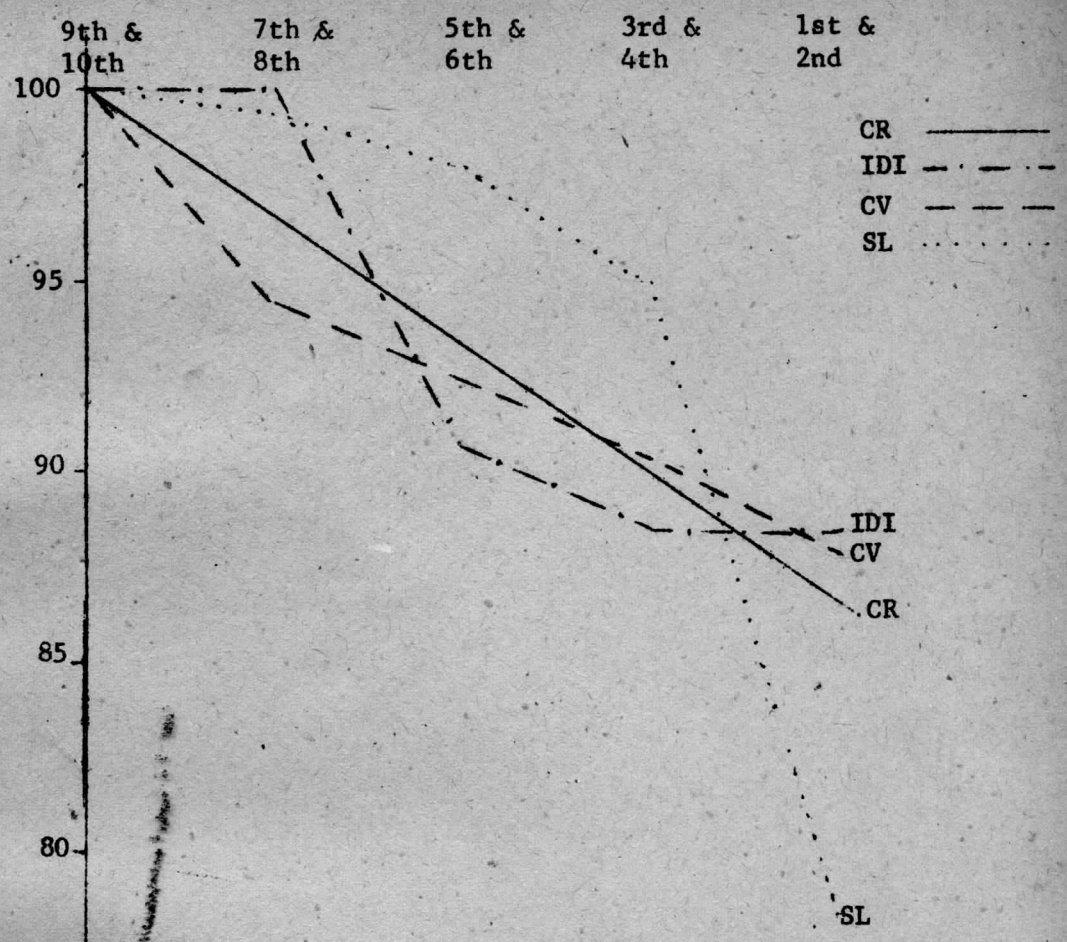


Chart 2

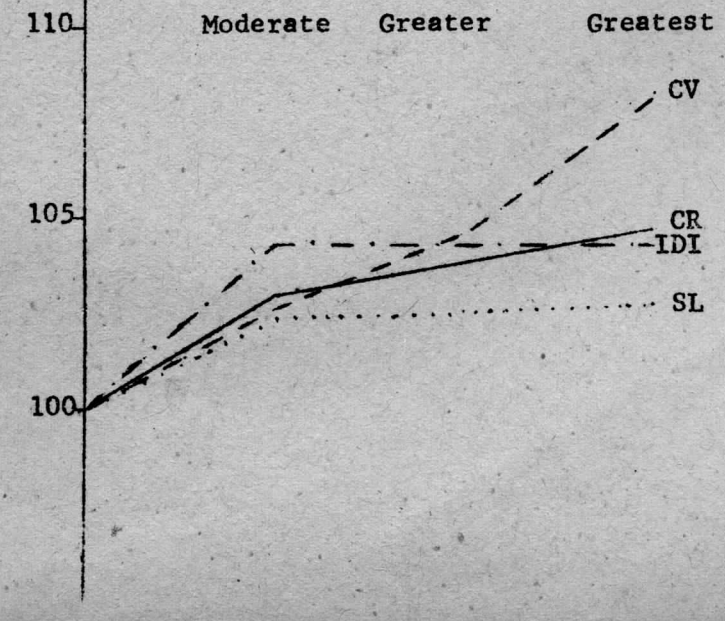

Chart 3

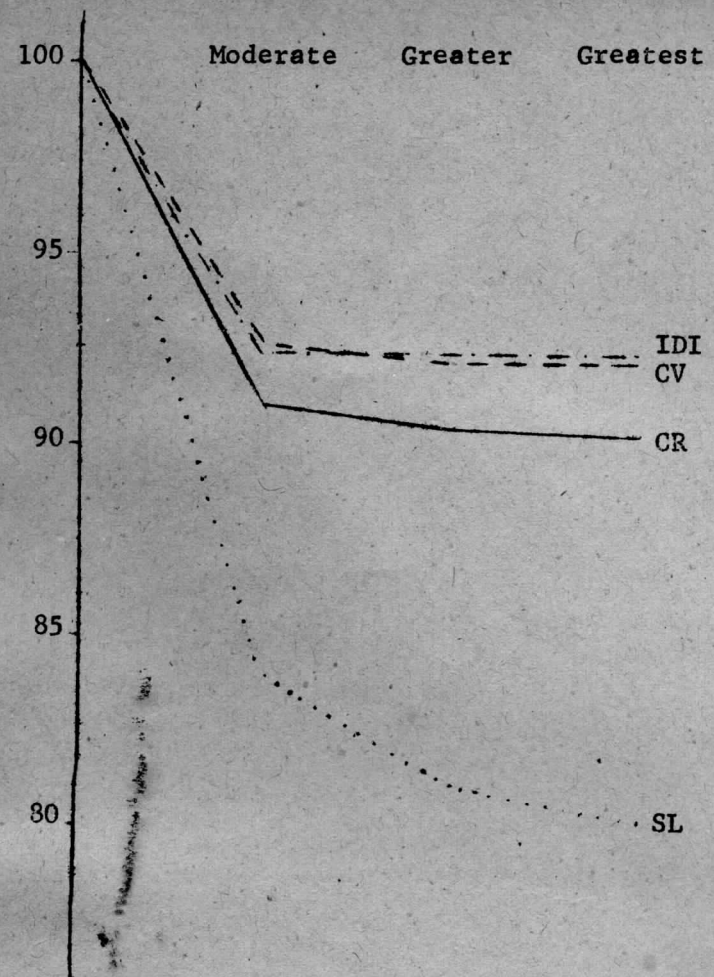


Chart 4