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**THE DYNAMICS OF INTERNATIONAL CAPITAL FLOWS
AND INTERNAL AND EXTERNAL BALANCE**

by

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The Dynamics of International Capital Flows and Internal and External Balance

S. C. Tsiang

I Introduction

Since Mundell's two seminal papers on the appropriate policy assignment for achieving internal as well as external balance appeared in 1960 and 1962,^{1/} a spate of literature on this topic has appeared in the journals of economics. Mundell's papers, however, contained a serious weakness which was unfortunately shared by too many writers who contributed to the discussion after him^{2/}; viz., the assumption that international capital flows can be treated as a function of the level of the domestic interest rate compared with foreign interest rates. Recent portfolio allocation theory, however, would clearly indicate that it is the changes in the interest differential between the domestic and foreign money markets that is responsible for the volatile capital flow between nations, rather than the size of differential. In other words, capital flows are to a large extent, adjustments of capital stocks held in different financial centers in response to changes in the yields at those centers. Such adjustments would come to an end after a period of time, if the yields at differential centers stay constant, even though at different levels.^{3/}

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Although Mundell recognized by way of an obiter dictum that capital flows might arise on account of stock adjustments, he lightly dismissed the fact by asserting that "it would not change the conclusions, although it may affect the quantitative extent of the policy changes required".^{4/}

The purpose of this paper is to point out that the conclusions which Mundell obtained would indeed be substantially altered as the substitution of a function for capital flows properly formulated to reflect the essential character of portfolio reallocation would substantially change the dynamic nature of the problem of achieving simultaneous internal and external balance.

In the comparative static (or comparative steady state) analyses of the problem, e.g., those contributed by M. Fleming, H. G. Johnson, R. W. Jones, J. Levin,^{5/} etc., where the stock adjustments are explicitly neglected as being transitory in nature and thus unnecessary to be taken into consideration in the steady state balance of payments, it may be shown that the exclusion of stock adjustments would drastically reduce the net effect of monetary policy on the balance of payments and might easily turn the net effect into a negative one. Furthermore, in considering the stability of the economic system under the policy mix suggested,

the capital flows due to stock adjustments certainly cannot be neglected; for once we are off the equilibrium point, capital stock adjustments would come into their own, perhaps even to dominate the scene. Unless the dynamic influences of capital flows due to stock adjustments are properly understood, we cannot safely prescribe the policy mix for attaining simultaneous internal and external balance from an initial position of disequilibrium.

II Formulation of an appropriate function of international capital flow

In this section we shall attempt to formulate a theoretically adequate yet sufficiently simple function of net international capital flow (outflow +, inflow-) that would include both transitory stock adjustments and sustainable flows. To simplify the problem, we shall assume that there are only two countries, the home country and the foreign country, and two kinds of assets, domestic assets and foreign assets. We shall start with a rather simple portfolio theory and assume that the residents of the home country would allocate a greater proportion of their total wealth to holdings of foreign assets if the interest rate on foreign assets rises relatively to domestic assets. Similarly, foreign residents would allocate a greater proportion of their wealth to their holdings of domestic assets if domestic interest rate rises relatively to their interest rate.

Thus the net capital position of the home country K (credit +, debit -) at equilibrium would be

$$(1) \quad K = \phi_d(r_f - r_d) W_d - \phi_f(r_d - r_f) W_f$$

where ϕ is the proportion of their total wealth W which the residents of a given country want to invest abroad. The subscripts d and f refer to the home and the foreign country, respectively. r_d and r_f are respectively, the interest rates in the two countries specified by the subscripts. Both ϕ_d and ϕ_f are assumed to be positive.

The equilibrium position, however, would not be reached instantaneously from any off-equilibrium initial position, nor would it be restored immediately if it is disrupted by exogenous changes in the determining arguments or in the parameters. Let us assume a simple exponential lag for the adjustments of both domestic and foreign portfolio allocation and for simplicity let us assume that the speed of adjustments in these two cases are the same. Thus

$$(2) \quad k = \lambda [\phi_d(r_f - r_d)W_d - \phi_f(r_d - r_f)W_f - K],$$

where $k = \dot{K}$ is the net capital outflow (inflow takes a minus sign) from the home country. If all the arguments and parameters

remain constant for a considerable length of time, k would eventually taper off to zero. Net capital flow, however, can be prevented from disappearing by constant or frequent changes in the arguments (neglecting changes in parameters for the time being). Differentiating the equation k with respect to time, we get

$$(3) \quad \dot{k} = \lambda[(\phi'_d W_d + \phi'_f W_f)(\dot{r}_f - \dot{r}_d) + \phi_d \dot{W}_d - \phi_f \dot{W}_f - k]$$

This equation together with the previous one indicate that a change in the domestic or foreign interest rate would bring about a change in the capital flow, but if the rate remained constant at the new level after it changes, then gradually net capital flow will again taper off to zero. However, in a dynamic world where the domestic or foreign interest rates and aggregate domestic or foreign wealth would keep on changing, k need not approach zero eventually. In fact in a steady state equilibrium, although \dot{r}_d , \dot{r}_f and \dot{k} may be presumed to have approached zero, \dot{W}_d and \dot{W}_f would be still going on at the rates of savings of the two countries, i.e., $s_d Y_d$ and $s_f Y_f$, respectively, where s_d and s_f are the propensities to save, and Y_d and Y_f are the national incomes of the two countries, respectively.

By putting \dot{k} , \dot{r}_d and \dot{r}_f to zero in equation (3), we can thus obtain the equation for the steady state equilibrium capital outflow k

as follows:

$$(4) \quad \bar{K} = \phi_d(\bar{r}_f - r_d) \cdot s_d Y_d - \phi_f(r_d - r_f) \cdot s_f Y_f,$$

where the variables with a bar on top are their equilibrium values. Those economists who believe that international capital flow is a function of the relative levels of the domestic and foreign interest rates instead of their changes must be thinking in terms of the sustainable equilibrium flow as formulated in equation (4). If it is our purpose merely to determine the conditions for the steady state equilibrium of the balance of payments and the level of employment, then it is perfectly all right to incorporate equation (4) for \bar{K} into the equation stating the condition for the balance of payments equilibrium. From such steady state equations for both internal and external equilibria, one may determine the final equilibrium values (or the required values) of all the variables, including the instrument or policy valuable. This seems to be the approach adopted by Fleming, Johnson, Jones, Levin, etc. However, such comparative steady state approach is not of much use in determining the dynamic time path of the state variables. Indeed, if we start from a position of disequilibrium, we could not be sure whether, by changing the policy variables in one bold stroke to their required equilibrium values (assuming that these

required values are precisely estimable), we would in fact converge onto the equilibrium position or not. For this kind of analysis tells us nothing about the inherent stability of the system.

As far as Mundell's dynamic approach is concerned, there is simply no excuse for abandoning the dynamic equation for capital flow, i.e., equation (3), in favor of its steady state variant, equation (4), except in drawing the phase line (his FF line) for the balance of payments equilibrium. The neglect of the dynamic nature of capital flows would invalidate most of his conclusions about the dynamic time path toward equilibrium.

Furthermore, if we are interested only in the steady state situation and neglect completely the dynamic stock adjustments, we would find that the effects of interest policy on the balance of payments would be drastically lower than what most people are used to expect. Indeed, when we include in the balance of payments the net interest payments (or receipts) on foreign investments, it may turn out that a rise in the domestic interest rate would very likely have a negative effect on the balance of payments.

The net international interest payments on foreign investments, I , may be formulated roughly as $I = r_d \phi_f W_f - r_f \phi_d W_d$. The effect of a domestic interest change on the net international interest

payments would then be

$$(5) \quad \frac{\partial I}{\partial r_d} = \phi_f W_f + \phi'_f r_d W_f + \phi'_d r_f W_d > 0$$

Thus the net effect of a change in domestic interest rate on the steady state balance of payments (neglecting the indirect effects through the income level) would be

$$(6) \quad -\left(\frac{\partial I}{\partial r_d} + \frac{\partial \bar{k}}{\partial r_d}\right) = -\phi_f W_f - \phi'_f \left(r_d \frac{W_f}{Y_f} - s_f\right) Y_f - \phi'_d \left(r_f \frac{W_d}{Y_d} - s_d\right) Y_d,$$

where $\frac{\partial \bar{k}}{\partial r_d}$ is derived from equation (4). It is not difficult to see that this expression is most probably negative. For the second term would be positive or negative according as $s_f \gtrless r_d \frac{W_f}{Y_f}$. If the wealth income ratio in foreign country is, say, 4, and the domestic interest rate is, say, 6%, then the foreign propensity to save would have to be greater than 24% to make this term positive. Similar considerations apply to the third term, except that the propensity to save and the wealth income ratio referred to there are those of the home country whereas the interest rate is that of the foreign country. Moreover, looming large over these two terms, which are presumably not very big whether positive or negative, is the first term which is always negative and likely to be much greater in absolute magnitude.

Thus the popular presumption that interest rate policy is

highly effective in improving the balance of payments, held even by those economists, who choose to neglect the transitory stock adjustments in their analysis, must be based upon a confusion of the transitory capital flows with the sustainable improvement in the balance of payments. This confusion, as will be shown presently in the next section, would profoundly affect the validity of the conclusions on appropriate policy assignment, as the induced stock adjustments and the sustainable capital flows play quite different roles in the dynamic mechanism of the economy.

III Reappraisal of Mundell's assignment problem ✓

Mundell in his 1962 article made the now well-known suggestion that to achieve both internal stability and balance of payments equilibrium in a country, which considers it inadvisable to alter the exchange rate or to impose trade controls, monetary policy should be assigned to achieve external balance and fiscal policy to achieve internal balance.^{6/}

His argument is based upon the stability analysis of the two differential equations purporting to describe the adjustment of the interest rate r (treated as the monetary policy instrument) and the budget surplus G (the fiscal policy instrument). ✓

If the monetary policy instrument r (the interest rate) is assigned to achieve balance of payments equilibrium and the fiscal policy instrument G (the budget surplus) is assigned to achieve internal balance, then the two dynamic differential equations would be

$$(7) \quad \begin{aligned} \dot{r} &= -k_1 B(r, G) \\ \dot{G} &= k_2 D(r, G) \end{aligned}$$

where B is the balance of payments surplus function, and D is the domestic excess demand function, k_1 and k_2 are the speed of adjustment coefficients, both defined as positive. B_r is assumed to be positive and so is B_G . D_r and D_G are both assumed negative. Furthermore, because Mundell has completely neglected the effect which a change in interest rate might have on the net international interest payments, he presumes that generally $\frac{B_r}{B_G} > \frac{D_r}{D_G}$, and hence $B_G D_r > B_r D_G$.

This dynamic system is globally as well as locally stable as the sufficient conditions for global stability (or the Olech conditions)^{7/}

- a) $-k_1 B_r + k_2 D_G < 0$ everywhere,
 - (8) b) $k_1 k_2 (B_G D_r - B_r D_G) > 0$ everywhere,
 - c) either $B_G D_r \neq 0$ everywhere, or $B_r D_G \neq 0$ everywhere.
- are expected to be satisfied. Thus no matter where we start from, by following this policy assignment, we may eventually arrive at a

joint equilibrium.

On the other hand, if the monetary policy instrument is assigned for internal balance and the fiscal instrument for external balance, then the pair of dynamic equations would be

$$(9) \quad \begin{aligned} \dot{r} &= h_1 D(r, G) \\ \dot{G} &= -h_2 B(r, G) \end{aligned}$$

where h_1 and h_2 are the respective speed of adjustment coefficients.

The Olech conditions for global stability would be

$$(10) \quad \begin{aligned} a) & \quad h_1 D_r - h_2 B_G < 0 \text{ everywhere,} \\ b) & \quad h_1 h_2 (B_r D_G - D_r B_G) > 0 \text{ everywhere,} \\ c) & \quad \text{either } B_r D_G \neq 0 \text{ everywhere, or } B_G D_r \neq 0 \text{ everywhere,} \end{aligned}$$

In this case, although the first and third conditions would still be satisfied, but the second one would be contradicted by his presumption that a change in interest rate has a favorable effect on the equilibrium balance of payments. Thus even local stability would be impossible. The second policy assignment, therefore, would not enable us to converge onto the simultaneous equilibrium point.

Mundell's analysis summarized above obviously failed to make any distinction between transitory capital flows due to portfolio adjustments and sustainable capital flows due to portfolio growth. It also erroneously presumed that the sustainable

effect on the balance of payments of a rise in interest rate would necessarily be large and favorable. In fact, the dynamic nature of international capital flows is totally overlooked.

To introduce the dynamic equation for capital movement into the analysis, however, would necessarily raise the order of the differential equation system to three and thus exclude the use of the popular two dimensional phase diagram to demonstrate the convergence to or divergence from the steady state equilibrium. However, as a Chinese poet statesman of old^{8/} used to say, it would be unwise to pare down our feet to fit the shoes. Nor is it advisable to oversimplify the real problem we are confronted with in order to suit our theoretical tools on hand.

With Mundell's first assignment scheme, i.e., fiscal instrument for internal balance and monetary instrument for external balance, we now have the following three differential equations:

$$\begin{aligned} \dot{G} &= \lambda_1 D(G, r) = \lambda_1 [Y(G, r) - Y_0] \\ (11) \quad \dot{r} &= \lambda_2 B(G, r, k) = \lambda_2 [M(Y) + I + k - X] \\ \dot{k} &= \lambda_3 [-\phi \dot{r} + k^* - k] = \lambda_3 [-\phi \dot{r} + \phi_d s_d Y - \phi_f s_f Y_f - k] \end{aligned}$$

where Y is the actual national income and Y_0 the target national income, $M(Y)$ the imports, I the net international interest payments as defined above, k the current net capital outflow as defined in

equation (2), and the third equation of (11) is equation (3) above with the following notation changes:—(i) $(\phi'_d W_d - \phi'_f W_f)$ is now abbreviated as ϕ' ; (ii) \dot{r}_d is simply written as \dot{r} , as only the variations of domestic interest rate are to be considered, whereas \dot{r}_f is assumed to be zero, (iii) $(\phi_d \dot{W}_d - \phi_f \dot{W}_f)$, the sustainable part of capital flows due to portfolio growth is written as $k^* = (\phi_d s_d Y_d - \phi_f s_f Y_f)$; (iv) λ , the speed of adjustment for international portfolios, is now given a subscript 3; and (v) the subscript is dropped for domestic income Y_d .

The characteristic equation of the linearized version of this system is

$$(12) \quad \begin{vmatrix} \lambda_1 Y_G - p & \lambda_1 Y_r & 0 \\ \lambda_2 m Y_G & \lambda_2 [m Y_r + \frac{\partial I}{\partial r}] - p & \lambda_2 \\ \lambda_3 \phi_d s_d Y_G & \lambda_3 [\phi_d s_d Y_r + \frac{\partial k^*}{\partial r} - \phi' p] & -\lambda_3 - p \end{vmatrix} = p^3 + a_1 p^2 + a_2 p + a_3 = 0$$

$$\begin{aligned} \text{where } a_1 &= \lambda_3 [(1 + \lambda_2 \phi') - \lambda_2 (m Y_r + \frac{\partial I}{\partial r}) - \lambda_1 Y_G] \\ a_2 &= \lambda_1 \lambda_2 \lambda_3 [(\frac{1}{\lambda_3} Y_G - \frac{1}{\lambda_1}) \frac{\partial I}{\partial r} - (\frac{1}{\lambda_2} + \phi') Y_G - \frac{1}{\lambda_1} (m Y_r + \frac{\partial k^*}{\partial r})] \\ a_3 &= \lambda_1 \lambda_2 \lambda_3 (\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r}) Y_G \end{aligned}$$

The Routh-Hurwitz necessary and sufficient conditions for local stability in this case are

(i) $a_1 > 0$, $a_2 > 0$, and $a_3 > 0$

(ii) $a_1 a_2 - a_3 > 0$.

Of the component terms in a_1 , only $-\lambda_2 \frac{\partial I}{\partial r}$ is negative, as Y_r and Y_G , which are known to be negative, both have a minus sign. Therefore, a_1 will be positive or negative according as

$$\lambda_3(1 + \lambda_2 \phi) - \lambda_2 m Y_r - \lambda_1 Y_G \gtrless \lambda_2 \frac{\partial I}{\partial r}.$$

Among the component terms of a_2 , again it is the term with $\frac{\partial I}{\partial r}$ that is negative, since $\frac{\partial k^*}{\partial r} = -(\phi_d' s_d Y_d + \phi_f' s_f Y_f) < 0$ by (5).

Thus a_2 will be positive or negative according as

$$-[(\frac{1}{\lambda_2} + \phi_f') Y_G + \frac{1}{\lambda_1} (m Y_r + \frac{\partial k^*}{\partial r})] \gtrless (\frac{1}{\lambda_1} - \frac{1}{\lambda_3} Y_G) \frac{\partial I}{\partial r}.$$

With Y_G known to be negative, a_3 will be positive if and only if $\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r} < 0$. This is however, very unlikely to be the case, as we have shown above in our discussion of equation (6) that

$$\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r} = \phi_f W_f + \phi_f (r_d \frac{W_f}{Y_f} - s_f) Y_f + \phi_d (r_f \frac{W_d}{Y_d} - s_d) Y_d,$$

which is very probably positive. Thus to follow Mundell's suggestion of assigning monetary policy to achieving external equilibrium and fiscal policy to achieving internal equilibrium would be unstable because the favorable effect of a rise in interest rate upon sustainable international capital flows is likely to be more than

offset by its unfavorable effect on international interest payments. The fact that a change in the domestic interest rate might also bring about large transitory capital flows due to portfolio readjustments does not help at all in remedying instability on this score. For although a large $\phi = (\phi_f W_f + \phi_d W_d)$, i.e., a large sensitivity of portfolio adjustments to interest change, would help to ensure that the coefficients a_1 and a_2 will be positive, it would be of no help at all in determining the sign of a_3 . Thus Mundell's assertion that the stock adjustment nature of international capital movement would not change the substance of his conclusions but merely affect the quantitative extent of the policy changes required is quite unwarranted.

It is of great interest to note that nor would the alternative scheme necessarily enable us to converge on the simultaneous internal and external equilibrium. Suppose we switch the policy assignment around such that

$$\begin{aligned} \dot{G} &= \lambda_1 [M(Y) + I + k - X] \\ (13) \quad \dot{r} &= \lambda_2 [Y(G, r) - Y_0] \\ \dot{k} &= \lambda_3 [-\phi \dot{r} + k^* - k] = \lambda_3 [-\phi \dot{r} + \phi_d s_d Y - \phi_f s_f Y_f - k] \end{aligned}$$

The characteristic equation of this differential equation system would be

$$(14) \begin{vmatrix} \lambda_1 m Y_G - p & \lambda_1 (m Y_r + \frac{\partial I}{\partial r}) & 1 \\ \lambda_2 Y_G & \lambda_2 Y_r - p & 0 \\ \lambda_3 \phi_d^s Y_G & \lambda_3 (\phi_d^s Y_r + \frac{\partial k^*}{\partial r} - \phi' p) & -\lambda_3 - p \end{vmatrix}$$

$$= p^3 + b_1 p^2 + b_2 p + b_3 = 0$$

where

$$b_1 = (\lambda_3 - \lambda_2 Y_r - \lambda_1 m Y_G) > 0$$

$$b_2 = \lambda_1 \lambda_2 \lambda_3 \left\{ \phi Y_G - \left[\frac{1}{\lambda_3} \frac{\partial I}{\partial r} + \frac{1}{\lambda_2} (\phi_d^s + m) \right] Y_G - \frac{1}{\lambda_1} Y_r \right\}$$

$$b_3 = -\lambda_1 \lambda_2 \lambda_3 \left[\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r} \right] Y_G$$

The Routh-Hurwitz conditions for local stability are

(i) $b_1 > 0$, $b_2 > 0$, and $b_3 > 0$;

(ii) $b_1 b_2 - b_3 > 0$.

It may be noticed immediately that b_3 here has the opposite sign of a_3 in equation (12), the characteristic equation of the previous case. Thus if $\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r} > 0$, causing instability in the previous case, b_3 would be positive, as required by the stability conditions.

However, we are not in the clear yet. If ϕ' , the interest sensitivity of stock adjustments is very large as generally believed to be the case between convertible countries, and if λ_3 ,

the speed coefficient of stock adjustments is also fairly large, then we might still be in trouble. For $b_2 \geq 0$, according as

$$\phi \leq \left[\frac{1}{\lambda_3} \frac{\partial I}{\partial r} + \frac{1}{\lambda_2} (\phi_d^s + m) \right] + \frac{1}{\lambda_1} \frac{Y_r}{Y_G}.$$

If ϕ as well as λ_3 are too large, b_2 might be negative. Thus capital flows due to stock adjustments and the sustainable capital flows due to domestic and foreign savings cannot be lumped together. They play entirely different roles in the dynamic mechanism.

In particular, when stock adjustments are highly sensitive to interest change whereas the sustainable capital flows are less responsive to interest change than international interest payments so that

$$\phi > \left[\frac{1}{\lambda_3} \frac{\partial I}{\partial r} + \frac{1}{\lambda_2} (\phi_d^s + m) \right] + \frac{1}{\lambda_1} \frac{Y_r}{Y_G},$$

and

$$\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r} > 0,$$

then neither the first policy assignment nor the second would converge on the joint equilibrium. In such cases, one obvious thing to try would be to stick to the second assignment scheme but to modify the definition of the balance of payments such that it includes only the sustainable capital flows but excludes the transitory stock adjustment flows. Thus our three differential

equations become

$$\begin{aligned}
 \dot{G} &= \lambda_1 [M(Y) + I + k^* - X] = \lambda_1 [M(Y) + I + \phi_d s_d Y - \phi_f s_f Y_f - X] \\
 (15) \quad \dot{r} &= \lambda_2 [Y(G, r) - Y_0] \\
 \dot{k} &= \lambda_3 [-\phi' \dot{r} + \phi_d s_d Y - \phi_f s_f Y_f - k]
 \end{aligned}$$

The characteristic matrix of this system would be

$$\begin{pmatrix}
 \lambda_1 (m + \phi_d s_d) Y_G - p & \lambda_1 (m Y_r + \frac{\partial I}{\partial r} + \phi_d s_d Y_r + \frac{\partial k^*}{\partial r}) & 0 \\
 \lambda_2 Y_G & \lambda_2 Y_r - p & 0 \\
 \lambda_3 \phi_d s_d Y_G & \lambda_3 (\phi_d s_d Y_r + \frac{\partial k^*}{\partial r} - \phi' p) & -\lambda_3 p
 \end{pmatrix}$$

which is obviously decomposable. The dynamic equation for \dot{k} would have no part to play in determining the dynamic path of G and r . For the latter variables would form a determinate system by themselves, the characteristic equation of which is

$$(16) \quad p^2 - [\lambda_1 (m + \phi_d s_d) Y_G + \lambda_2 Y_r] p + \lambda_1 \lambda_2 (\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r}) = 0$$

The stability conditions are satisfied since

$$\begin{aligned}
 &\text{and} \\
 (a) \quad &\lambda_1 (m + \phi_d s_d) Y_G + \lambda_2 Y_r < 0 \\
 (b) \quad &\lambda_1 \lambda_2 (\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r}) > 0.
 \end{aligned}$$

Thus if we are to assign a particular policy for maintaining balance of payments equilibrium, that policy should be the fiscal policy rather than the monetary policy. Furthermore, as

the indicator for fiscal policy, the balance of payments must be re-defined to exclude the transitory capital flows due to stock adjustments so that fiscal policy might not be led on a wild goose chase after these highly volatile transitory elements.

IV Further Look into the Basic Stability of the System

Mundell's approach to the problem of achieving simultaneous internal and external equilibrium is unsatisfactory in yet another sense. He treats the policy variables as the dynamic variables, whose movements over time are governed by the differential equations under investigation, whereas other variables, e. g., Y and M are all assumed to adjust themselves instantaneously to policy variables in a stable manner. Actually policy adjustments are seldom, if ever, continuous but rather are carried out intermittently at fairly wide time intervals, whereas other economic variables generally adjust themselves continuously with distributed lags of different lengths to the discrete changes of policy variables, and their dynamic stability cannot always be taken for granted. His differential equation system, therefore, does not seem to give a realistic picture of the working of our economic system.

What appears to be a more realistic approach is first to

test whether the system under perfectly free operation without any government intervention (in a sort of simulated gold standard operation) would be stable or not. If its stability is assured, then we may try to determine from the estimated structural equations of the economy the approximate values of the policy variables consistent with the desired equilibrium situation by the usual comparative static (steady state) method as suggested by Fleming, Johnson and Jones, and then change these policy variables accordingly in one bold stroke. If our quantitative knowledge of the structure equations is not sufficient to determine precisely the final required values of the policy variables, then the method of assigning one policy instrument to one target on the basis of the principle of comparative effectiveness, i. e., the principle of effective market classification, as suggested by Mundell could be tried in order to reach the joint equilibrium step by step.

Should the system under perfectly free operation be found unstable, it would be extremely dangerous to attempt either of these two ways of achieving the joint equilibrium. From the analysis of stability, however, we may perhaps be able to determine which market reaction function should be constrained by astute government intervention so as to yield a stable system.

To illustrate my point, let us suppose that we are dealing with the following free operating system:

$$\begin{aligned}
 \dot{Y} &= \lambda_1 [E(Y, r) - M(Y) + X - Y] \\
 \dot{r} &= \lambda_2 [L(Y, r) - (\theta R + N)] \\
 \dot{k} &= \lambda_3 [-\phi_r^* + (\phi_d s_d Y - \phi_f s_f Y) - k] \\
 \dot{R} &= [X - M(Y) - k - I]
 \end{aligned}
 \tag{17}$$

when $E(\cdot)$ is the domestic expenditure function; R is the foreign exchange reserves of the monetary authorities and θ the money supply multiplier applicable to the reserve basis; N is the fiduciary part of money supply treated here as an exogenous parameter.

The characteristic equation of this system would be

$$\begin{aligned}
 (18) \quad & \begin{vmatrix} \lambda_1(E_y - m - 1) - p & \lambda_1 E_r & 0 & 0 \\ \lambda_2 L_y & \lambda_2 L_r - p & 0 & -\lambda_2 \theta \\ \lambda_3 \phi_d s_d & \lambda_3 \left(\frac{\partial k^*}{\partial r} - \phi^* p \right) & -\lambda_3 - p & 0 \\ -m & -\frac{\partial I}{\partial r} & -1 & -p \end{vmatrix} \\
 & = p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4 = 0
 \end{aligned}$$

where $a_1 = [\lambda_3 - \lambda_2 L_r - \lambda_1 (E_y - m - 1)]$

$$a_2 = \lambda_1 \lambda_2 \lambda_3 \left[(E_y - m - 1) \left(\frac{L_r}{\lambda_3} - \frac{1}{\lambda_2} \right) - \left(\frac{L_r}{\lambda_1} + \frac{L_y E_r}{\lambda_3} \right) + \frac{\theta}{\lambda_1} \left(\phi_r^* - \frac{1}{\lambda_3} \frac{\partial I}{\partial r} \right) \right]$$

$$a_3 = \lambda_1 \lambda_2 \lambda_3 \left\{ (E_y - m - 1) \left[L_r - \theta \left(\phi_r^* - \frac{1}{\lambda_3} \frac{\partial I}{\partial r} \right) \right] - \theta \left[\frac{1}{\lambda_1} \frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r} + \frac{m E_r}{\lambda_3} \right] - L_y E_r \right\}$$

$$a_4 = \lambda_1 \lambda_2 \lambda_3 \theta \left[(E_y - m - 1) \left(\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r} \right) - E_r (\phi_d s_d + m) \right]$$

The Routh-Hurwitz conditions for local instability of the equilibrium point are:

$$(i) \quad a_1 > 0, a_2 > 0, a_3 > 0 \text{ and } a_4 > 0.$$

$$(ii) \quad a_1 a_2 - a_3 > 0.$$

$$(iii) \quad a_1 a_2 a_3 - a_3^2 - a_1^2 a_4 > 0.$$

It is not difficult to see that some of the necessary stability conditions might fail to be satisfied. E_y , the marginal propensity to spend is usually taken as smaller than unity. But in time of full (or nearly full) utilization of capital capacity, when increase in aggregate demand is likely to stimulate new investment activities (the acceleration principle), it can obviously exceed unity (or even $1 + m$) by a considerable margin. If $(E_y - m - 1)$ is positive, then a_3 could become negative and upset stability. For if ϕ' , the interest sensitivity of international portfolio adjustments, is very large as is generally assumed to be the case between convertible countries, and if the speed of portfolio adjustments λ_3 is also very great, then $(\phi' - \frac{1}{\lambda_3} \frac{\partial I}{\partial r})$ is likely to be positive and large. The only positive terms in a_3 will be $-(\theta \frac{m}{\lambda_3} + L_y)E_r$. If

$$(19) \quad (\theta \frac{m}{\lambda_3} + L_y) |E_r| < \{ (E_y - m - 1) [\theta (\phi' - \frac{1}{\lambda_3} \frac{\partial I}{\partial r}) + |L_r|] + \frac{\theta}{\lambda_1} (\frac{\partial I}{\partial r} + \frac{\partial \kappa}{\partial r}) \},$$

the system will be unstable. It is particularly likely to be the case, if $|L_r|$, the interest sensitivity of the demand for money, is

already very large.

This is not surprising at all. Intuitively, this may be grasped in the following way. We know that if $(E_y - m - 1)$ is positive, the Hicksian IS curve would slope upward and, therefore, the economy would not be stable with what Meade calls a "Keynesian neutral monetary policy," i.e., the monetary policy that pegs the domestic interest rate at a given level with infinitely elastic money supply.^{9/} Now if capital flows are infinitely elastic with respect to interest rate, and if money supply is linked to capital flows through the effect of the latter on monetary reserve basis, then we will have in effect a "Keynesian neutral monetary policy" pegging the domestic interest rate at the level of foreign interest rate. The resulting instability would not, therefore, be unexpected.

On the other hand, if $(E_y - m - 1)$ is negative, there is also a slight possibility of instability. That is, if

$$(20) \quad |E_r| (\phi_d s_d + m) < (m + 1 - E_y) \left(\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r} \right)$$

a_4 would be negative. This possibility, however, is very remote. For although we have seen above that $\frac{\partial I}{\partial r} + \frac{\partial k^*}{\partial r}$ is very probable positive and quite large too, yet E_r , the effect of a change in

interest rate on aggregate domestic expenditure is presumably numerically much larger.

Anyway, it seems definitely inadvisable to follow the gold standard rules of the game when international capital movement are known to be highly interest-sensitive due to quick portfolio adjustments. It would seem advisable to sever or at least to constrain the link between domestic money supply and the balance of payments, 10/

This may be carried out in various degrees of thoroughness, e.g., (1) To sterilize all balance of payments deficits or surplus completely, thus severing all links between balance of payments and money supply. (2) To sterilize all capital account deficits or surpluses and net interest payments, but maintain a link between trade balance and money supply. (3) To sterilize all capital account balance, but maintain a link between current account balance (i. e., trade balance and net interest payments) and money supply.

Sterilization policy (1) virtually means put θ to zero in the second differential equation of (17). The characteristic matrix of the system shown in (18) would have three consecutive zeros in the fourth column. The matrix clearly becomes decomposable. The first two equations would form a determinate system by themselves,

the necessary and sufficient stability conditions would be

$$(a) \quad \lambda_1(E_y - m - 1) + \lambda_2 L_r < 0$$

$$(b) \quad \lambda_1 \lambda_2 [(E_y - m - 1) L_r - E_r L_y] > 0$$

If $(E_y - m - 1) < 0$, these conditions are certainly always satisfied. If $(E_y - m - 1) > 0$, as can very well be the case when the degree of capital utilization is high in the economy, then two precautions should be taken to ensure stability. First, the speed of adjustment of the money market (the speed of response of the interest rate) must not be allowed to be too sluggish, so as to ensure that the first condition will always be satisfied. That is, we must first ensure that

$$\lambda_2 > \frac{\lambda_1(E_y - m - 1)}{|L_r|}$$

Secondly, we must see to it that

$$|L_r| < \frac{|E_r| \cdot L_y}{(E_y - m - 1)}$$

That is to say, we must not let the demand for money become overly sensitive to interest rate changes. Since the demand for money generally has a tendency to become highly interest sensitive when the interest rate falls to a very low level, as is shown by the "inverse J" shaped demand curve for money obtained by many

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economists in the past, ^{11/} therefore, it implies that extreme forms of cheap money policy (or policy of super-abundant liquidity) should definitely be shunned. ^{12/}

Provided these two precautions are taken, the above stability conditions can generally be satisfied without much difficulty.

If the sterilization policy (2) is adopted, the fourth equation of (17) is in effect changed to

$$\dot{R}^* = [X - M(Y)].$$

Again the characteristic matrix of (18) would become decomposable, as the third column, which already has two zeros on top, would now have another zero at the bottom. k in fact would have no feedback on the other three variables. The characteristic equation is now

$$\begin{vmatrix} \lambda_1(E_y - m - 1) - p & \lambda_1 E_r & 0 \\ \lambda_2 L_y & \lambda_2 L_r - p & -\lambda_2 \theta \\ -m & 0 & -p \end{vmatrix} = p^3 + a_1 p^2 + a_2 p + a_3 = 0$$

$$\text{where } a_1 = -[\lambda_1(E_y - m - 1) + \lambda_2 L_r]$$

$$a_2 = \lambda_1 \lambda_2 [(E_y - m - 1)L_r - E_r L_y]$$

$$a_3 = -\lambda_1 \lambda_2 \theta E_r m$$

The necessary and sufficient conditions for local stability would be

$$(a) a_1 > 0, \quad (b) a_2 > 0, \quad (c) a_3 > 0, \quad (d) a_1 a_2 - a_3 > 0$$

The first two are in fact the same as the two conditions in the preceding case. The third condition presents no problem as $E_r < 0$, and $m > 0$. In effect, this case requires only one additional condition that

$$-[\lambda_1(E_y - m - 1) + \lambda_2 L_r][(E_y - m - 1)L_r - E_r L_y] + \theta E_r m > 0.$$

which does not seem to present much difficulty when other conditions are satisfied.

To adopt the third sterilization policy would modify the fourth equation of (17) to

$$\dot{R}^{**} = [X - M(Y) - I].$$

The characteristic equation of the system would now become

$$\begin{vmatrix} \lambda_1(E_y - m - 1) - p & \lambda_1 E_r & 0 \\ \lambda_2 L_y & \lambda_2 L_r - p & -\lambda_2 \theta \\ -m & -\frac{\partial I}{\partial r} & -p \end{vmatrix} = p^3 + a_1 p^2 + a_2 p + a_3 = 0$$

$$\text{where } a_1 = -[\lambda_1(E_y - m - 1) + \lambda_2 L_r]$$

$$a_2 = \lambda_1 \lambda_2 [(E_y - m - 1) L_r - E_r L_y - \frac{\theta}{\lambda_1} \frac{\partial I}{\partial r}]$$

$$a_3 = \lambda_1 \lambda_2 [(E_y - m - 1) \frac{\partial I}{\partial r} - E_r m] \theta$$

The Routh-Hurwitz conditions are

$$(a) \ a_1 > 0, \quad (b) \ a_2 > 0, \quad (c) \ a_3 > 0, \quad (d) \ a_1 a_2 - a_3 > 0.$$

In this case, the first condition remains the same as in the preceding case. But the second condition becomes less favorable as a negative terms $-\frac{\theta}{\lambda_1} \frac{\partial I}{\partial r}$ is added into a_2 . For the third condition that $a_3 > 0$, the addition of the term $(E_y - m - 1) \frac{\partial I}{\partial r}$ would be unfavorable to stability if $(E_y - m - 1)$ is negative. It seems, therefore, that, on the whole, not to sterilize the net international payments of interests and let them exert an influence on the money supply would be unfavorable to stability, although its unfavorable influence is presumably not big enough to upset the stability of the system, as might be the case, if volatile capital flows due to portfolio adjustments are not sterilized.

Anyway, the presumption of traditional economists that the gold standard rules of the game are necessarily conducive to the equilibrium of the world economy is certainly open to serious doubts. Our preliminary analysis here rather indicates that not

only should the domestic money supply not be linked rigidly to neither the full balance of payments deficit or surplus but also that the money supply nor the interest rate should be used as an instrument to bring about the balance of payments equilibrium either.

Footnotes

^{1/}Robert A. Mundell, "The Monetary Dynamics of International Adjustment Under Fixed and Flexible Exchange Rates," Quarterly Journal of Economics, LXXIV, May 1960, 227-257; and "The Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability," I M F Staff Papers, March 1962, 70-79.

^{2/}E.g., J. Marcus Fleming, "Domestic Financial Policies Under Fixed and Floating Exchange Rates," I M F Staff Papers, IX, November 1962, 369-379; Harry G. Johnson, "Some Aspects of the Theory of Economic Policy in a World of Capital Mobility," in T. Bagioti (ed.), Essays in Honour of Marco Fanno, Padua: Cedam, 1966; Ronald W. Jones, "Monetary and Fiscal Policy for an Economy with Fixed Exchange Rates," Journal of Political Economy, 76, July/August 1968, 921-943; etc.

^{3/}This fact has been most forcibly pointed out by Willet and Forte. See Thomas D. Willet and Francisco Forte, "Interest Rate Policy and External Balance," Quarterly Journal of Economics, LXXXIII, May 1969, 242-262, and also Willet, "A Portfolio Theory of International Short Term Capital Movements," unpublished Ph.D. thesis, University of Virginia, 1967. Willet and Forte, however, failed to formulate a dynamic equation for capital flows and thus was not able to discuss rigorously the different roles played by capital flows due to stock adjustments and capital flows

due to portfolio growth (sustainable flows) in the dynamic mechanism of an open economic system, which we shall attempt here.

The present author has long ago emphasized the portfolio adjustment nature of capital flows in his previous articles, "A Theory of Foreign Exchange Speculation Under a Floating Exchange System," Journal of Political Economy, LXVI, Oct. 1958, 399-418; and "The Theory of Forward Exchange and Effects of Government Intervention on the Forward Exchange Market," I M F Staff Papers, April 1959, 75-106.

4/"The Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability," op. cit., p. 71, fn. 2.

5/Fleming, op. cit.; Johnson, op. cit.; Jones, op. cit.; and Jay H. Levin, "International Capital Mobility and the Assignment Problem," Working Papers, No. 11, Department of Economics, Cornell University, April 1971.

6/"The Appropriate Use of Monetary and Fiscal Policy," op. cit.

7/C. Olech, "On the Global Stability of an Autonomous System on the Contributions to Differential Equations, Vol. I, (1963), 389-400.

8/Su Shih (1036-1101 A.D.) of Sung Dynasty, also known as Su Tung-Po.

9/J. E. Meade, The Theory of International Economic Policy, Vol. I,

The Balance of Payments, Mathematical Supplement, London, 1951, pp. 31 and 49.

^{10/} In fact, Mundell's treatment of the domestic interest rate as a policy instrument, which the authorities can set at any level they wish, implies total sterilization of balance of payments surplus or deficit.

^{11/} For the case of the United States, see for instance J. Tobin, "Liquidity-Preference and Monetary Policy," Review of Economic Statistics, XXIX, February 1947, 124-131; and M. Bronfenbrenner and T. Mayer, "Liquidity Functions in the American Economy," Econometrica, XXVIII, October 1960, 810-834. For the case of the United Kingdom, see for example A. J. Brown, "Interest, Prices, and the Demand Schedule for Idle Money," Oxford Economic Papers, No. 2, May 1939, 46-69; and A. M. Khusro, "Investigation of Liquidity Preference," Yorkshire Bulletin of Economic and Social Research, IV, January 1952, 1-20.

^{12/} This point has been made by the present author previously in criticism of Milton Friedman's concept of the optimum supply of money. See S. C. Tsiang, "A Critical Note on the Optimum Supply of Money," Journal of Money, Credit, and Banking, I, May 1969, 266-280.

