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INTEREST RATE AND CONSUMPTION: A COMMENT

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Birdsall and Goldstein's paper makes the net effect of interest rate changes on consumption and savings appear extremely indeterminate and uncertain.[1/] This appearance of indeterminacy, however, is partly due to the formulation of their model, which assumed away a very important factor that would have changed the picture, and partly because of their attempt to generalize their analysis to cover all complicated patterns of expected changes in future interest rates. If we reintroduce the neglected factor and concentrate on the classical pattern of interest change, viz., a change in current interest rate expected to be permanent, it would then be clear that the effect upon current consumption and saving is by no means so indeterminate as Birdsall and Goldstein make it out to be.

Specifically, Birdsall and Goldstein assume in their paper that the initial assets of each individual (or household) consist entirely of assets of one period duration, the rates of return on which are recontracted anew at the beginning of each period.[2/] In this way, they ruled out the possibility of appreciation or depreciation of existing assets induced by changes in interest rates.

Yet a major proportion of non-human assets in any community must consist of such durable assets as land, durable structures, equity shares and bonds, that would appreciate or depreciate with expected changes in interest rates. To emphasize how recognition of this fact

would affect their conclusions, let us assume that the initial wealth of a typical individual consists, instead of one period assets envisaged by Birdsall and Goldstein, but of assets expected to yield a permanent streams of income equal to z_o. If the current market interest rate i is expected to be permanent, their current market value must be equal to $Z = \frac{z_o}{i}$. Subsequent investments of his new savings, however, would fetch a yield no greater or smaller than the prevailing market rate of interest. As the current market rate is always expected to be permanent, the wealth constraint of the individual concerned (who is expected to dispose of all his net worth at the end of his planning horizon) can be written as

$$(1) \qquad \frac{z_o}{i} + \sum_{k=1}^{n} (x_k + z_o - c_k)(1 + i)^{n-k} = 0,$$

where x_k is his non-asset income expected for the period k, c_k in his planned consumption for period k, n is the last period of his planning horizon, and c_n is supposed to include bequest.

The Slutzky equation for the effects of a permanent change in the current interest rate on current consumption would be

$$(2) \qquad \frac{\partial c_1}{\partial i} = -\gamma \sum_{k=2}^{n} (k-1)(1+i)^{-k} \frac{D_{k1}}{D} + (1+i)^{-n} \frac{D_{o1}}{D} \{ \frac{\partial}{\partial i}(\frac{z_o}{i}) + \sum_{k=1}^{n} (n-k)$$
$$(x_k + z_o - c_k)(1+i)^{n-k-1} \},$$

where γ is the Lagrange multiplier of the wealth constraint, D is the bordered Hessian of order n + 1, and D_{k1} and D_{o1} are, respectively the cofactors of the terms in the corresponding rows and the first column inside the border of D.

The first sum represents the ~~net compensated substitution effect~~. Since according to Hicks' "third rule for substitution terms"[3/]

$$-\sum_{k=2}^{n}(1+i)^{k-1}\frac{D_{k1}}{D}=\frac{D_{11}}{D}<0,$$

substitutability ~~between consumptions of different periods~~ (positive $\frac{D_{k1}}{D}$'s) must dominate complementarity (negative $\frac{D_{k1}}{D}$'s). Thus, even if we do not rule out complementarity as Birdsall and Goldstein did, the compensated substitution effect ~~must be negative~~, unless the weighting factor (k - 1) somehow by a strange conincidence tips the balance. In other words, unless future consumptions complementary to current consumption c_1 are predominantly of very distant future periods and, hence, are given progressively greater weights (a very unlikely event), the net substitution effect must be negative.

The terms inside the square brackets represents the ~~wealth and income~~ effects. The coefficient $\frac{D_{o1}}{D}$ may be presumed positive, since current consumption is most unlikely to be an inferior good. The summation

$$(1+i)^{-n}\frac{D_{o1}}{D}\sum_{k=1}^{n}(n-k)(x_k+z_o-c_k)(1+i)^{n-k-1}$$

corresponds to what Birdsall and Goldstein call "the ~~life-time income effect~~", which they claim to be so indeterminate as to make it impossible to say anything definite about the effects of a change in interest rate. However, if we recall the wealth constraint, it is clear that

$$\sum_{k=1}^{n}(x_k-z_o-c_k)(1+i)^{n-k}=-\frac{z_o}{i},$$

and hence must be negative, if the individual concerned has any positive

initial wealth. And an average individual must have some positive initial assets, since a typical community must have some net non-human assets (including land) in existence at any time. Thus the "life time income effect" must be negative, unless the weighting factor (n - k), which gives progressively greater weights to periods close to the present, tips the scale in the opposite direction. In othe other words, unless the typical individual saves significant positive amounts out of his incomes in a sufficient number of early periods and dissaves mainly in later periods close the end of his life period, the life-time income effect would only reinforce the negative substitution effect.[4/]

We have not yet reckoned with the term

$$(1+i)^{-n} \frac{D_{o1}}{D} \frac{\partial}{\partial i} \left(\frac{z_o}{i}\right) = -(1+i)^{-n} \frac{D_{o1}}{D} \frac{z_o}{i^2},$$

which neither Birdsall and Goldstein, nor Yaari, nor even Feldstein and myself took into account.[5/] This term we may call the "interest-induced wealth effect" to distinguish from the "life time income effect". It is definitely negative and thus would reinforce the compensated substitution effect. Some idea of its relative magnitude compared with the "life-time income effect" can be obtained by recalling again that by the wealth constraint

$$\sum_{k=1}^{n} (x_k + z_o - c_k)(1 + i)^{n-k} = - z_o/i,$$

whereas

$$\frac{\partial}{\partial i} \left(\frac{z_o}{i}\right) = - z_o/i^2.$$

If i is around 5%, the difference between the two is by a factor of 20.

Thus even if the "life-time income effect" is positive because the individual is an early-period saver, it could be easily offset by the negative "interest induced wealth effect."[6/]

FOOTNOTES

1/ William C. Birdsall and Jon H. Goldstein, "The Effect of Interest Rate Changes on Consumption Allocation over Time," current issue of this journal.

2/ Yaari, who analyzed the same problem using a continuous model, and whose work was quoted extensively by Birdsall and Goldstein, also made the assumption that all assets bear the ruling market rate of interest at every instant of time, thus excluding the possibility of interest-induced appreciation and depreciation of existing assts.
See. M.E. Yaari, "On the Consumer's Lifetime Allocation Process," International Economic Review, V, Sept. 1964, pp. 304-317.

3/ J.R. Hicks, Value and Capital, 2nd Edition, Mathematical Appendix, p. 311.

4/ All this is in close accord with the conclusions Feldstein and myself obtained from a simple two-period model. There we concluded that if the individual is a dis-saver in the first period, his saving would definitely increase with the interest rate. Only when the individual is a saver in the first period and dissaver in the second, would the substitution and income effects operate in opposite directions, but it is still uncertain which will be the dominant force.
See M. Feldstein and Tsiang, "The Interest Rate, Taxation and the Personal Savings Incentive," Quarterly Journal of Economics, LXXXII, August 1968, pp. 419-434.

5/ However, in my earlier article, "A Model of Growth in Restovian Stages," Econometrica, 32, Oct. 1964, pp. 619-48 , it was clearly pointed out that there is an important interest induced wealth effect in addition to the substitution and income effects," p. 625 fn. 8.

6/ Even such an avowed Keynesian as Warren L. Smith has now admitted that the interest-induced wealth effect constitutes an important effect of monetary policy. See Smith "On Some Current Issues in Monetary Economics: An Interpretation," Journal of Economic Literature, Sept. 1970, pp. 767-782, esp. 770.
In econometric studies of the effect of interest rates on consumption and savings, however, the interest-induced wealth effect has generally been overlooked, as wealth is generally treated as a separate independent variable along with the interest rate. Thus the interest-induced changes in the current value of existing wealth are treated as exogenous changes and not imputed to the interest changes that caused them. See e.g., C. Wright, "Some Evidence on the Interest Elasticity of Consumption," American Economic Review, LVII, Sept. 1967, pp. 850-855.
Wright also incorrectly considered the implicit "life-time income effect" as being included in the regression coefficient of consumption on explicit income variable. Actually the "life-time income effect" of a change in interest rate, being implicit and not measurable as any explicit income changes, must be reflected in the regression coefficient of consumption on interest rate if it is reflected anywhere at all.