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DYNAMICS OF AN AGRARIAN MODEL WITH Z-GOODS*

by

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economy open to trade.[2] The Hymer-Resnick model is dynamized by incorporating agricultural capital, which is taken to be the cumulated portion of Z-output allocated in the past for investment. In consumption Z-goods are also assumed to be substitutable to industrial consumption goods. What emerges is a growth model of the agrarian economy differing from previous theoretical models in many respects, as will be evident below.

Formal specification of the model is given in Section I. In Section II it is shown how equilibrium is attained in the short run and in the long run. Section III describes the pattern of labor allocation, output, consumption and trade as the agrarian economy develops over time. Section IV analyzes the short-run and long-run effects of a change in the terms of trade on the endogenous variables of the model, making comparison with the partial equilibrium analysis of Hymer and Resnick and illustrating the possibility of opposing qualitative results of comparative statics and dynamics. Finally, in Section V, it is argued that the agrarian economy does not necessarily have to fall into a low-level equilibrium in the long run, in contrast with the generally pessimistic prognosis of previous agrarian models.

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[2] One may think of the export economies of Southeast Asia during the period from the opening of the Suez Canal to the Second World War as historical examples (see Resnick). Undoubtedly, many parts of the contemporary underdeveloped world, especially the peasant economies of Africa, are in the same situation.

I. The Model

Following the Hymer-Resnick schema, two types of production activities are distinguished in the present model -- one producing the usual agricultural (consumption) goods F which may be sold to the world market in exchange for industrial consumption goods C and the other consisting of labor-using Z-activities which produce nontradable Z-goods that may either be consumed as an inferior substitute for C-goods or used to augment agricultural capital. For simplicity, production in both F and Z are assumed subject to constant returns to scale and unchanging technology.

Since labor is the only input in Z-production, we may write[3]

(1) $$x_Z = \alpha l_Z ,$$

where

x_Z = per capita output of Z-goods

l_Z = fraction of total labor engaged in Z-activities

α = (constant) average productivity of labor in Z-production.

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[3]The case of diminishing returns in Z-activities, i.e., $x_Z = g_Z(l_Z)$, $g''_Z<0$ is eschewed in this paper. In my dissertation (pp. 58-61) it is shown that diminishing returns can give rise to multiple long-run equilibria and dynamic instability of the system.

Production in F , which requires inputs of labor and agricultural capital, is also characterized by positive but diminishing marginal returns. Thus,

(2) $x_F = l_F g_F(\tilde{\rho})$ where $g_F' > 0$, $g_F'' < 0$ for all $\tilde{\rho}$, and

x_F = per capita output of F-goods

l_F = fraction of total labor engaged in F-production

$\tilde{\rho}$ = ratio of the stock of agricultural capital R to labor employed in F-production; a measure of capital intensity in producing F-goods.

Labor allocation in peasant agriculture may reasonably be assumed to equalize the value of marginal productivities in F- and Z-production,[4] i.e.,

(3) $$w \equiv g_F - \tilde{\rho} g_F' = \alpha p_Z \quad ,$$

where w and p_Z are the imputed (shadow) prices of labor and Z-goods, respectively -- each expressed in terms of F-goods.

Since labor has positive marginal returns in either activity, it will be fully employed:

(4) $$l_F + l_Z = 1.$$

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[4]Professor Schultz (Chap. 3) presents a strong case for the highly efficient manner in which resources are allocated even in traditional agriculture.

Concerning the division of newly-produced Z-goods into consumption and saving (investment),[5] we make the plausible assumption for peasant economies that the amoun of saving corresponds to some fraction of current income (output) irrespective of the imputed shares of factor earnings. Thus, given a consumption demand function for Z-goods, static equilibrium in the Z-market is described by

(5) $$q_F(y, p_Z, p_C) = x_Z - \frac{sy}{p_Z} ,$$

where

$g_Z(\ )$ = consumption demand for Z-goods

p_C = world price of C-goods relative to F

s = saving rate (assumed constant)

y = per capita total income (output) in terms of F.

Notice that

(6) $$y \equiv x_F + p_Z x_Z = w + \rho g_F' .$$

The second term in the second expression for **y** represents per capita imputed rent to agricultural capital, where ρ is ratio of the stock of capital to <u>total</u> labor.

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[5] An analytical similarity may be noted with one-sector growth models which also assume a single commodity functioning either as a consumption good or as a capital good; once invested however, it remains a capital good and cannot be used for consumption.

Noting the definition given earlier for δ , we may write

(7) $$\rho = \delta l_F \ .$$

Consumption demand for Z , as shown in the L.H.S. of (5) is assumed to depend on income, the price of Z-goods, and the price of C-goods. (Because of the small, open character of the agrarian economy envisaged in the model, p_C may be treated as a parameter.) A priori signs of the partial derivatives are given by

(8) $$\frac{\partial q_Z}{\partial y} < 0 \ , \ \frac{\partial q_Z}{\partial p_Z} < 0 \ \text{ and } \ \frac{\partial q_Z}{\partial p_C} > 0 \ ,$$

where the first inequality reflects the inferiority of Z-goods in consumption, the other two showing the directions of the own-price and cross-substitution effects.

Industrial consumption goods can only be imported, since the economy is purely agrarian. Assuming trade balance, imports of C-goods must equal the exportable surplus in F -- the only other tradable commodity. Specification of the demand function for C will thus complete the static model. We may write

(9) $$q_C(y, p_Z, p_C) = m_C \ ,$$

where m_C is the amount of C-imports (equal to F-exports) and $q_C(\ )$ is the consumption function for C with the following properties:

(10) $$\frac{\partial q_C}{\partial y} > 0 \ , \ \frac{\partial q_C}{\partial p_Z} > 0 \ \text{ and } \ \frac{\partial q_C}{\partial p_C} < 0 \ .$$

The dynamic assumptions of the model describe how the two factors -- labor and agricultural capital -- change over time. The labor force L is assumed to be identical to the size of population, which grows at a constant rate λ. Therefore the increment to the labor force per unit time is given by

(11) $$\dot{L} = \lambda L.$$

Assuming exponential decay of agricultural capital and using the saving assumption given earlier, net investment per capita may be written

(12) $$\frac{\dot{R}}{L} = \frac{sy}{p_Z} - \frac{\mu R}{L} \ ,$$

where μ is the rate of depreciation of capital.

The differential equations (11) and (12) may be collapsed into one:

(13) $$\frac{\dot{\rho}}{\rho} \equiv \frac{\dot{R}}{R} - \frac{\dot{L}}{L} = \frac{sy}{p_Z \rho} - (\mu + \lambda) \ ,$$

recalling that $\rho \equiv \frac{R}{L}$.

II. Determination of Short-Run and Long-Run Equilibria

At any moment of time, the stock of agricultural capital R, the labor force L (and hence ρ) as well as the world price of industrial consumption goods p_C are known. It follows that (1), (2), the second relation in (3), (4), (5), the first relation in (6) and (7) may be used to solve simultaneously for the seven variables β l_F, l_Z, x_F, x_Z, p_Z and y. The remaining variables w and m_C may then be determined from the first relation in (3) and (9), respectively -- completing the determination of short-run (static) equilibrium for the current period. Note finally that $\dot{\rho}$ can be obtained from (13) and hence the stage is set for the determination of a new short-run equilibrium for the next period.

Mere counting of variables and equations is not enough however to guarantee a unique solution to the equation system. To establish uniqueness of short-run equilibrium, each variable above must be shown to be uniquely determined by the agricultural capital-labor ratio. Here we simply state the following important result:[6] A sufficient (but not necessary) condition for the unique existence of short-run equilibrium in the agrarian economy is that the saving rate be greater than the

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[6] For the derivation, the interested reader is referred to the author's dissertation (pp. 54-55).

absolute value of the marginal propensity to consume Z-goods, i.e.,

$$s > \left|p_Z \frac{\partial q_Z}{\partial y}\right| \equiv |b_Z|. \tag{14}$$

If the parameters s and b_Z are such that inequality (14) is not satisfied, then it is possible to have multiple short-run equilibria. In other words, equilibrium in the Z-market given in (5) will be met at more than one point in the production possibility frontier, e.g., one where x_F is high and p_Z low and one where x_Z and p_Z are high, as illustrated in Figure 1.

Condition (14) above bears similarity to the "capital intensity condition" in the standard neoclassical two-sector growth model where the capital goods industry must be less capital intensive than the consumer goods industry for uniqueness of short-run equilibrium.[7] Notice however that the latter is a technological requirement, in contrast with the purely behavioral character of (14).

The growth of the economy consists of the sequence of short-run equilibria generated over time by the movement of the capital-labor ratio ρ which is itself determined endogenously in the dynamic model, given an initial value of ρ . Will the dynamic system converge to a unique long-run (dynamic) equilibrium?

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[7] See, for example, Uzawa and Solow.

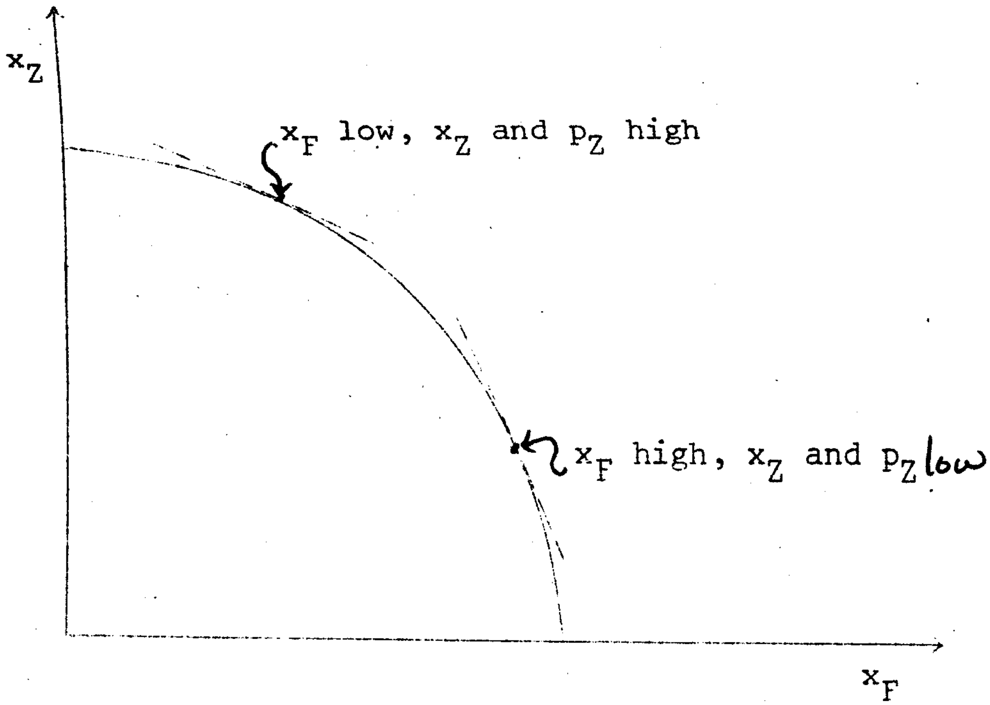

FIGURE 1

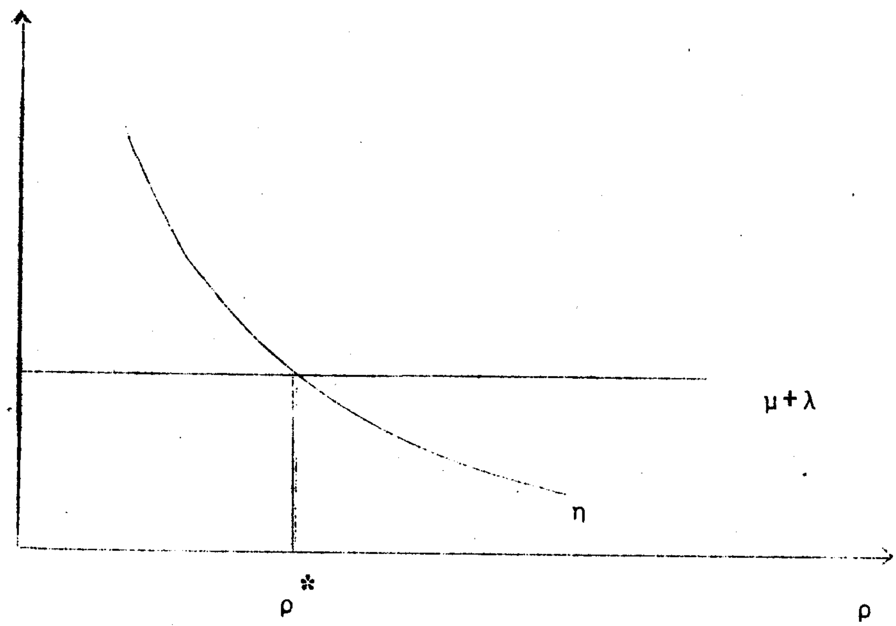

FIGURE 2

Let us assume that condition (14) is satisfied, ruling out therefore multiple long-run equilibria on account simply of the multiplicity of possible short-run equilibrium positions. In (13) let the rate of growth of undepreciated agricultural capital be defined by

(15) $$\eta \equiv \frac{sy}{\rho p_Z} .$$

Differentiation with respect to ρ yields, upon simplification,

(16) $$\frac{\rho p_Z}{s} \cdot \frac{d\eta}{d\rho} = \frac{-w}{\rho} + (y - l_Z w) \frac{g_F''}{wD} ,$$

where

(16 a) $$D \equiv \frac{w l_F}{\tilde{\rho}} + \frac{\tilde{\rho} g_F''}{\alpha} \left(p_Z \frac{\partial q_Z}{\partial p_Z} - \frac{sy}{p_Z}\right) > 0 .$$

It follows from (16) and (16a) that

(17) $$\frac{d\eta}{d\rho} < 0 \quad \text{for all } \rho ,$$

and hence there is at most one long-run equilibrium value of ρ, say ρ^*. The determination of ρ^* is illustrated in Figure 2. The curve labelled η shows the gross rate of growth of agricultural capital for any factor ratio and is negatively sloped everywhere, as required by (17). It is drawn asymptotic to both axes since $\lim_{\rho \to 0} \eta = \infty$ and $\lim_{\rho \to \infty} = 0$ are implied by the restrictions imposed earlier on the production functions. The other solid curve in Figure 2 shows a horizontal line representing the sum of the two parameters μ and λ. Clearly, the point of intersection of the two curves -- at which the long-run equilibrium condition $\eta = \mu+\lambda$ is satisfied -- is unique. Dynamic stability is also evident from the graphical construction. If capital is growing faster (slower) than labor, then there is a tendency to reduce (raise) the gross rate of growth of per capita capital until is equal to the sum of the depreciation rate and the rate of growth of labor.

At the long-run equilibrium point,

(18) $$\rho^* = \frac{sw}{G},$$

where

(18 a) $$G \equiv (\mu+\lambda)\, p_Z - sg_F' > 0 \quad \text{for all} \quad \rho^* > 0.$$

Equation (18) is obtained by setting $\frac{\dot{\rho}}{\rho}$ in (13) to zero and substituting the second expression for y in (6).

III. Development Path of the Agrarian Economy

It should follow from the foregoing discussion that, given a sufficiently low initial ratio of agricultural capital to labor, the economy will experience a continously increasing capital-labor ratio in its monotonic approach to the long-run equilibrium position. How will the other variables of the model, e.g., capital intensity in F-production, F- and Z-output levels, etc. change over time as the small agrarian economy open to trade develops, assuming constant terms of trade?

Let u denote any of these variables. Uniqueness of short-run equilibrium implies that u is expressible in terms of ρ ; hence

(19) $$\frac{du}{dt} = \frac{du}{d\rho} \cdot \frac{d\rho}{dt} ,$$

which relates the time derivatives of u and ρ. Since $\frac{d\rho}{dt} > 0$ as shown earlier, it suffices to evaluate the qualitative impact of a change in the capital-labor ratio on u in order to determine the direction of change of each variable u as the economy grows.

A change in ρ will disturb the static equilibrium in the economy. Differentiating with respect to ρ the equation system consisting of (1) - (7) and (9), and solving for $\frac{d\tilde{\rho}}{d\rho}$, we obtain

(20) $$\frac{d\tilde{\rho}}{d\rho} = \frac{1}{\tilde{\rho}D}\left[w + (b_Z+s)\tilde{\rho}g_F'\right]$$

where $D > 0$ is as defined in (16a). Thus if $s > |b_Z|$ as required by (14) , $\frac{d\tilde{\rho}}{d\rho} > 0$ for all ρ , so that the developing agrarian economy may be expected to exhibit an increasing capital intensity in F-production over time.

The variables ρ , $\tilde{\rho}$ and l_F are related definitionally and hence the effect of an increase in the capital-labor ratio on labor allocation may be determined by differentiating (7) with respect to ρ , from which

(21) $$\frac{\rho}{l_F} \frac{dl_F}{d\rho} = 1 - l_F \frac{d\tilde{\rho}}{d\rho} = \frac{-1}{D} \left[(b_Z+s)l_F g_F' + \frac{Bp_Z \tilde{\rho} g_F''}{w}\right]$$

where

(21a) $$B = -(p_Z \frac{\partial q_Z}{\partial p_Z} - \frac{sy}{p_Z}) > 0$$, noting the second relation in (8).

The two terms within the square brackets in the R.H.S. of (21) are opposite in sign; hence the elasticity of l_F with respect to ρ may be greater than or less than zero. It is seen that the proportion of labor time spent in F-production will more likely increase as the economy develops: (1) the more price elastic is the demand for Z-goods; (2) the greater the inferiority of Z-goods; and (3) the more strongly diminishing returns in F-production is operating. Since

(22) $$\frac{dx_F}{d\rho} = l_F g_F' \frac{d\tilde{\rho}}{d\rho} + g_F \frac{dl_F}{d\rho}$$ from (1) ,

the same statement applies to the impact on the output of F-goods. From (4) and (1), the reverse conditions hold in the case of p_Z and x_Z . It does not seem inevitable therefore that the output of Z-goods will diminish over time in the developing agrarian economy. Hence it does not follow that since much of Z-activities are not included in national income accounts the growth of G.N.P. necessarily overstates actual growth of output as claimed by Hymer and Resnick (p. 504 n).

A change in the capital-labor ratio will also be accompanied by a change in the valuation of Z-goods. Differentiating the second relation in (3) with respect to ρ gives

(23) $$\alpha \frac{dp_Z}{d\rho} = - g''_F \frac{d\tilde{\rho}}{d\rho} .$$

Since $\frac{d\tilde{\rho}}{d\rho} > 0$, the sign of $\frac{dp_Z}{d\rho}$ is unambiguously positive. The interesting result follows that, as the agrarian economy grows over time and accumulates agricultural capital at a faster rate than population growth, the imputed price of Z-goods increases.

Total output (income) should increase over time with the factor ratio. This may be verified by differentiating the second expression for y in (6) with respect to ρ and noting the first relation in (3) and (4):

(24) $$\frac{dy}{d\rho} = -\tilde{\rho} l_Z g_F'' \frac{d\tilde{\rho}}{d\rho} + g_F' > 0 .$$

Finally, we may examine how consumption of C- and Z-goods are affected by the increase in the capital-labor ratio concomitant with growth. The former would be identical to the effect on C-imports and also to the supply response in F-production (in terms of the exportable surplus) by assumption of trade balance. Noting (9) and (5),

(25)
$$\frac{dq_C}{d\rho} = \frac{\partial q_C}{\partial y} \cdot \frac{dy}{d\rho} + \frac{dp_Z}{d\rho} \cdot \frac{\partial q_C}{\partial p_Z}$$
$$\frac{dq_Z}{d\rho} = \frac{\partial q_Z}{\partial y} \cdot \frac{dy}{d\rho} + \frac{\partial q_Z}{\partial p_Z} \cdot \frac{dp_Z}{d\rho} .$$

Under the assumptions made earlier concerning the signs of the partial derivatives (see (10) and (8)) and recalling that $\frac{dy}{d\rho}$ and $\frac{dp_Z}{d\rho}$ are both greater than zero, consumption of industrial consumption goods will increase while that of Z-goods will decrease unambiguously, given that ρ rises over time. Notice from (25) that the cross-substitution effect of the induced rise in p_Z tends to reinforce the positive income effect in C-consumption and the negative income effect in Z-consumption.

IV. The Terms of Trade

The relationship between the terms of trade and economic growth of primary-producing countries is a subject widely investigated in the development literature. The present analysis differs from others in three significant respects: (1) a small country is explicitly considered; (2) a third commodity (Z-goods) is added to the usual classification of agricultural and industrial goods; and (3) a rigorously formulated dynamic model of the agrarian economy is employed. The first implies a one-way causation from an exogenous changes in the country's terms of trade to the induced changes in the other variables (instead of their mutual interaction in the two-country model). The second represents a more realistic view of the range of economic choice in both production and consumption relevant to present-day agrarian economies. The third constitutes a departure from the usual static framework used in previous investigations and makes possible an analytical distinction between the short-run and long-run effects of a change in the terms of trade on the endogenous variables of the model.

Consider first the short-run impact of a change in p_C. It is clear that static equilibrium will be disturbed. Differentiating the static model consisting of equations (1) - (7) and (9) with respect to p_C and solving for $\frac{d\tilde{p}}{dp_C}$, we get

(26) $$\frac{d\tilde{\rho}}{dp_C} = \frac{p_Z x_Z}{q_Z D} \cdot \frac{\partial q_Z}{\partial p_C} > 0 ,$$

noting (16a) and the third inequality in (8). Therefore, capital intensity in F-production will unambiguously rise, given a deterioration of the terms of trade of the agrarian economy.

Moreover,

$$\frac{dx_F}{dp_C} = - \frac{wl_F}{\tilde{\rho}} \cdot \frac{d\tilde{\rho}}{dp_C} < 0 \quad , \frac{dx_Z}{dp_C} = \frac{\alpha l_F}{\tilde{\rho}} \cdot \frac{d\tilde{\rho}}{dp_C} > 0 ,$$

(27) $$\frac{dp_Z}{dp_C} = - \frac{\tilde{\rho} g_F''}{\alpha} \cdot \frac{d\tilde{\rho}}{dp_C} > 0 \quad \text{, and hence}$$

$$\frac{dl_F}{dp_C} < 0 \quad , \frac{dl_Z}{dp_C} > 0 \quad , \frac{dy}{dp_C} = x_Z \frac{dp_Z}{dp_C} > 0$$

since $$\frac{dx_F}{dp_C} = p_Z \frac{dx_Z}{dp_C} .$$

Thus a rise in the world price of industrial consumption goods leads in the short run to a reallocation of labor from F- to Z-production, raising Z-output but depressing production of F. As shown above, the reduction in F-output is equal to the increment in the value of Z(expressed in terms of F). Because C and Z are substitutes in consumption, there is an induced rise in the price of Z-goods[8], causing the value of total

[8]It may be noted here that in the partial equilibrium framework used by Hymer and Resnick, p_Z is treated as a parameter and curiously remains unaffected by a change in p_C .

output to increase (a rather surprising result).

Let us now examine the short-run effects of a change in p_C on the current consumption of Z and C . Differentiating (5) with respect to p_C gives

$$\text{(28)} \qquad \frac{dq_Z}{dp_C} = \frac{dx_Z}{dp_C} - \frac{s}{p_Z}\frac{dy}{dp_C} + \frac{sy}{p^2}\frac{dp_Z}{dp_C}$$

$$= \frac{dx_Z}{dp_C} + \frac{sx_F}{p_Z^2}\frac{dp_Z}{dp_C}$$ by substituting the expression for $\frac{dy}{dp_C}$ in (27).

The sign of $\frac{dq_Z}{dp_C}$ in (28) is positive, noting (27); hence a rise in p_C will induce the agrarian economy to raise short-run consumption of Z regardless of its inferiority. Using (9),

$$\text{(29)} \qquad \frac{dq_C}{dp_C} = \frac{\partial q_C}{\partial y} \cdot \frac{dy}{dp_C} + \frac{p_C\, m_C}{p_Z} \cdot \frac{\partial q_Z}{\partial p_C} \cdot \frac{dp_Z}{dp_C} + \frac{\partial q_C}{\partial p_C} .$$

Consumption of C-goods (equivalently, importation of C or exportable surplus in F) may increase or decrease in the short run as p_C rises depending on the relative magnitudes of the terms in the R.H.S. of (29). The first term represents the income effect and is positive. The two remaining terms are price (substitution) effects which are opposite in sign. The second term, which may be labelled the cross-price effect[9], is always positive since $\frac{\partial q_Z}{\partial p_C}$ and $\frac{dp_Z}{dp_C}$ are both greater than zero; the own price effect, represented by the third term, is negative. Thus, only if industrial consumption goods were "sufficiently" elastic (i.e., the own-price effect dominates the combined income and cross-price effects) would the usual negatively sloped demand curve hold true. In terms of supply responsiveness, such condition would insure a normally behaved supply function: since q_C also represents the exportable surplus in F , $\frac{dq_C}{dp_C} < 0$ implies that an increase in relative price of F-goods in the world market will give rise to a higher surplus in F for exportation.

Turning now to the determination of the long-run effects of a change in the price of industrial consumption goods, we may first differentiate with respect to p_C the long-run equilibrium equation (18) to obtain

$$(30) \qquad G \frac{d\rho}{dp_C} + (\mu+\lambda)\rho\frac{dp_Z}{dp_C} + sl_Z\rho g''_F\frac{d\tilde{\rho}}{dp_C} = 0 .$$

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[9]The cross-price effect does not enter in the Hymer-Resnick analysis of supply responsiveness in the agrarian economy because of the assumed fixity of p_Z .

A second relationship among the derivatives terms may be obtained from the equilibrium requirement in the market for Z-goods[10]:

(31)
$$-B\frac{dp_Z}{dp_C} + (b_Z+s)l_F g_F' \frac{d\tilde{\rho}}{dp_C} + \left[w + (b_Z+s)\tilde{\rho} g_F' \right] \frac{dl_F}{dp_C} = \frac{-p_Z}{q_Z} \cdot \frac{\partial q_z}{\partial p_C} ,$$

where B is as defined in (21a) .

Moreover, from (7) and the second relation relation in (3),

(32)
$$\frac{d\rho}{dp_C} - l_F \frac{d\tilde{\rho}}{dp_C} - \tilde{\rho} \frac{dl_F}{dp_C} = 0,$$

$$\frac{dp_Z}{dp_C} = \frac{-\tilde{\rho} g_F''}{\alpha} \cdot \frac{d\tilde{\rho}}{dp_C} .$$

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[10]Equation (31) is obtained by differentiating (5) with respect to p_C and substituting

$$\frac{dx_F}{dp_C} = l_F g_F' \frac{d\tilde{\rho}}{dp_C} + g_F \frac{dl_F}{dp_C}$$ from (2) and

$$\frac{dx_Z}{dp_C} = \alpha \frac{dl_Z}{dp_C} = -\alpha \frac{dl_F}{dp_C}$$ using (1) and (4) .

Equations (30) - (32) can be solved for the four derivative terms. In particular,[11]

$$\text{(33)} \qquad \frac{d\rho}{dp_C} = p_Z \frac{\partial x_Z}{\partial p_C} \cdot \frac{s x_F \tilde{\rho}^2 g_F''}{D_o}$$

where

$$D_o = Gw\left(l_F w - \frac{B\tilde{\rho}}{\alpha} g_F''\right) - s x_F \tilde{\rho} g_F'' \left[w + (b_Z + s)\tilde{\rho} g_F' \right].$$

Given the assumptions concerning signs made earlier, $\frac{d\rho}{dp_C}$ in (33) is always less than zero. It follows that a deterioration in the terms of trade of the agrarian economy will lead in the long run to a lowering of the equilibrium factor ratio ρ^*.

Having obtained the impact of a change in p_C on the equilibrium value of ρ, the results of the preceding section may be used to derive the effects on the remaining variables. Thus,

$$\text{(34)} \qquad \frac{d\tilde{\rho}}{dp_C} = \left.\frac{\partial \tilde{\rho}}{\partial \rho}\right|_{p_C = \text{const.}} \cdot \frac{d\rho}{dp_C} < 0 \quad \text{by (20) and (33),}$$

$$\text{(35)} \qquad \frac{dl_F}{dp_C} = \left.\frac{\partial l_F}{\partial \rho}\right|_{p_C = \text{const.}} \cdot \frac{d\rho}{dp_C} > 0 \quad \text{by (21) and (33).}$$

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[11] Equation (33) is derived by applying Cramer's rule to the matrix equation in (32) after dp_Z/dp_C has been eliminated in (30) and (31); D_o in the denominator of the R.H.S. of (33) is the determinant of the resulting transformation matrix.

From (34), a rise in the world price of industrial consumption goods will result in a lower agricultural capital intensity in F-production in the new long-run equilibrium. The qualitative result of comparative statics, it may be recalled, suggests otherwise. The impact on labor allocation is seen from (35) to be uncertain; in contrast, the earlier short-run analysis shows that l_F will decrease unambiguously.

A change in the price of C-goods will also influence the equilibrium value of total output. Observing that

$$(36) \qquad \frac{dy}{dp_C} = \left.\frac{\partial y}{\partial \rho}\right|_{p_C=\text{const.}} \cdot \frac{d\rho}{dp_C}$$

and recalling that $\left.\frac{\partial}{\partial \rho}\right|_{p_C=\text{const.}} > 0$ from (24), it is seen

that the value of total output (in terms of F-goods) will decrease with an increase in p_C . Earlier, the paradoxical result was obtained that in the short run a deterioration of the terms of trade will raise total output in the agrarian economy because of the induced rise in the price of Z-goods. The present result shows that eventually the economy can be expected to exhibit a lower value of total output (with the computed price of Z-goods also reduced).

Consumption of Z-goods in the new long-run equilibrium will change with p_C as indicated in (28). Since the sign of $\frac{dx_Z}{dp_C}$ is ambiguous in the present case, q_Z may increase or decrease in the long run, given that p_C increases. In the short run, however, it was demonstrated that Z-consumption must increase.

Finally, it may inferred from (29) and later results that long-run imports of industrial consumption goods will decrease unambiguously with a deterioration of the terms of trade, the income and substitution effects being mutually reinforcing. Equivalently, exportable surplus will diminish in the long run. The short run effect, as shown earlier, is ambiguous.

Table 1 summarizes the qualitative effects of a change in

	C. S.	C. D.
$\tilde{\rho}$	+	--
$l_F, -l_Z$	−	±
$x_F, -x_Z$	−	±
p_Z	+	−
y	+	−
q_Z	−	±
q_C, m_C	±	=

Table 1

p_C for both the comparative statics (C.S.) and comparative dynamics (C.D.) of the model. The divergence of results for the two cases points to the possibility that what is true in the short run may not hold over a longer period of time.

V. Stagnation or Steady Growth?

Most development theorists share an essentially pessimistic view of the long-term growth performance of agrarian economies. A general tendency of the economy to stagnate in the long run represents a common feature of the agrarian models used -- as surveyed, for example, by Fei and Ranis.

The model of the small, open agrarian economy presented above also points to a long-run tendency toward a unique equilibrium (under certain sufficient conditions) in which per capita output remains unchanged. But whether such long run position in the present context should be labelled a state of stagnation remains an open question. If the agrarian economy happens to be in long-run equilibrium at a high level of per capita output, it can hardly be called stagnating; steady growth seems a more appropriate description.

It is in fact possible for the agrarian economy in our model to maintain a high level of output per head in the long run. For one thing, openness of the small agrarian economy makes such long-run position dependent on the prevailing world price of agricultural goods relative to that of industrial consumption goods. As shown in the preceding section, a higher long-run value of per capita output accrues from a higher terms of trade of the agrarian economy. But more at the heart of the matter is the crucial role of two parameters of the model, *viz.*, the saving rate and the population growth rate. It seems intuitively reasonable (and is in fact a well-known proposition in neoclassical growth theory) that a higher output per capita in the long run will result from either a rise in the saving rate or a lowering of the population growth or a lowering of the population growth rate. (This can be demonstrated rigorously by the method of comparative dynamics, as in the analysis of a change in the terms of trade done in the preceding section.) While most other dynamic models of the agrarian economy contain an exogenous population growth rate, they do not have a place for a saving-investment parameter because capital is not entered explicitly as an input in agricultural production. In the model discussed above, Z-goods production makes possible a choice between consumption and investment to confront the peasant family. Such economic choice is absent in the simple framework of the monolithic agrarian economy, ruling out in effect an important basis for a brighter long-run prospect of the economy.

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