

at year's end is a determinant of output.

The remaining equations appear to require no comment. We merely note that the variable  $A$ , net factor income from abroad, is a residual item and plays no real role in the model.

The model is consistent in that there are just enough equations to give the values of the endogenous variables. There are no two ways of determining real output, unlike "two-gap" models where the difference between domestic investment and saving,  $I - (Y - C)$ , is in general not equal to the net inflow of foreign capital,  $M - X - A$ . In the present case, these two are always equal because of the way  $A$  is calculated. One disadvantage here, of course, is that predictions of  $A$  are likely to be poor; but this would seem to be a minor disadvantage. The more important point is that the model does not take into account possible constraints on the net inflow of foreign capital: it implicitly assumes that the necessary inflow is always forthcoming. This aspect of the problem should be considered in making projections.

We have specified output as largely supply-determined. We say "largely" because there are also elements of demand implicit in the determination of output in the model. For changes in the money supply which affect output through the price level and employment, are of course determined by the government deficit, net private credit creation and the balance of payments surplus, all of which reflect demand considerations. These variables are absent in the basic model but will be included in the government, monetary and foreign trade submodels planned for future work. This leads us to say a word about the exogenous variables.

It is clear that since exports affect the balance of payments, and  $X$  are not causally independent. That they are exogenous variables simply means that they are unexplained in the model;<sup>7/</sup> in a larger and more complete model, they would be classified as endogenous. Thus, when we say that the effect of an increase in exports is not on the current year's output but on the following year's (through eqs. (4) and (5)), the implicit assumption would be that the policymaker's maintain the money supply constant by reducing credit the same amount as the increase in export proceeds; otherwise a passive policy would evidently entail a larger money supply and therefore higher output the same year. Analogous remarks apply to the other exogenous variables.

### 3. Reduced-Form Equations

Eqs. (1) - (3) constitute a simultaneous equation system in  $Y$ ,  $N$  and  $P$  that can be easily solved in terms of the predetermined variables  $K$ ,  $W$ , and  $Z$ . We have;

$$(1') \quad Y = 5249.6 + .4066 K - 5.009 W + 1.084 Z$$

$$(2') \quad N = 8921.9 + .2161 K - 5.223 W + 1.130 Z$$

$$(3') \quad P = 62.849 - .0017 K + .0215 W + .0376 Z$$

Looking at these equations, we see the separate effects of  $K$ ,  $W$  and  $Z$  on  $Y$ ,  $N$  and  $P$ . An increase in the capital stock of \$100 million raises output by \$40 million (both at 1955 prices) and employment by nearly 22 thousand persons, and the price index decreases by 0.17. A \$100 increase in the annual money wage rate reduces output by half a billion pesos and employment by over half a million persons, and raises the



price index over 2 points. A £100 million increase in the money supply increases output by £108 million and employment by 113 thousand, but it also increases the price level by almost 4 index points. There is here a necessary policy choice.

The other reduced-form equations can be obtained using eqs. (1') - (5') in a straightforward fashion, but the non-linearities make them too long and relatively uninteresting to reproduce here. We report, however, a few reduced-form coefficients which may be of some interest.

The coefficient of  $Z$  in the equation for  $M$  is  $.0959 + 78.06/P_m$ . With  $P_m$  equal to 250 this implies that a £100 million increase in  $Z$  increases imports (at 1955 prices) by about £40 million, which would be £100 million at current prices. Attempts to raise output and employment by rapidly increasing the money supply could thus be frustrated by a balance of payments constraints.

In the equation for  $I$ ,  $Z$  has a coefficient of  $.5471 + 25.61/P_m$ . Thus a £100 million increase in  $Z$  increases real investment by about £76 million, which at current prices would exceed £130 million. An examination of the calculation of the  $Z$ -coefficient shows that about two-thirds of its magnitude are associated with the variable  $P$ , i.e., the impact of the money supply on investment is caused mainly through the price level.

The last point to consider is the coefficient of  $K$  in the equation for  $I$ ,  $.0647 - 1.185/P_m$  which we shall denote by  $\alpha$ . This would seem to imply that if all the exogenous variables are held constant, the capital stock would still grow (eventually) at about 6% a year, in which case the model has no stationary state solution and is explosive. However, it should be

recalled that we have defined  $K$  in terms of cumulated gross investment. If we want to consider the long-run implications of the model- which would be a theoretical exercise more than anything else- it would be necessary to redefine  $K$  in eq. (10) so as to take account of physical depreciation, e.g.  $K = (1-d)K_{-1} + I_{-1}$  where  $d$  is the annual percentage rate of depreciation (assuming "radioactive decay"). This is equivalent to retaining eq. (10) but replacing  $a$  with  $(a-d)$  in the reduced-form equation for  $I$ . In this case, with  $a < d$ , there would be a stationary state solution (see Appendix 1). It should be emphasized, however, that this point is worth only passing mention for empirical models like the present one.

In regard to stability, the more interesting question is whether or not, given the values of the exogenous variables for (say) 1955-1969,<sup>8/</sup> the model "predicts" values of the endogenous variables that tend to be close to what was observed. The value of  $K$  for 1955 would be needed of course to start off this experiment, but  $K$  for later years would be automatically generated. An increasing divergence of the generated time-path of an endogenous variable from the actual time-path would indicate some systematic error in the model. The results of this experiment show no such divergence; on the contrary, there is rather a tendency toward convergence.

It is to be noted that 1955 was an atypical year: both  $P_m$  and  $P_x$  were at their lowest that year, and excepting 1954 (when  $P$  had a value of 99.4),  $P$  was also at its lowest. As may be expected, the model's solution for 1955 gives values of the endogenous variables relatively far from observations. Convergence towards actual time-paths from such a starting point thus seems to indicate some kind of stability in the model.

#### 4. Dynamic Multipliers

The preceding section has considered only the impact effects of changes in the predetermined variables. Some interest may also attach to the consequences beyond the first year of an exogenous disturbance. So-called dynamic multipliers provide quantitative information on the effects over a specified period on the endogenous variables of the model. In this section we investigate the dynamic multipliers associated with the following exogenous variables of the model: Z, W, and X.

It is common practice to consider an initial situation of long-run equilibrium when the exogenous disturbance is applied. The dynamic multipliers are then evaluated in terms of the deviation of the generated path from the initial equilibrium. Here, we examine the deviation from the path of the economy (not necessarily in equilibrium in the initial period) when undisturbed by a change in the exogenous variable. This is clearly the more relevant information for policy purposes since the economy in actuality never attains long-run equilibrium.

Substituting the definitional expression for  $K$  of eq. (10) in the reduced-form equations  $(1')-(3')$ , we find that investment  $I_{t-1}$  and capital stock  $K_{t-1}$  lagged one year are determinants of output  $Y$ , employment  $N$  and the price level  $P$ . Investment in turn is determined partly by the level of importation, from eq. (5). Hence, in evaluating the dynamic effects on the economy of an exogenous stimulus it is necessary to use the import equation (4) and the investment function (5) in conjunction with (10),  $(1')$ ,  $(2')$  and  $(3')$ .



The reduced-form equation for  $I$ , assuming  $P_m = 250$ , is given by

$$(5') \quad I = 588.9 + .0600 K + .6495 Z - 1.8335 \overset{70}{\downarrow} + .1163 X + 2.3768 P_x$$

In order to calculate dynamic multipliers, it would be necessary to allow for depreciation (cf. Section 3). Assuming, for simplicity, a half-life of 10 years, we need only to subtract the implied depreciation rate (.0670) from the coefficient of  $K$  in eq. (5') to be able to interpret  $I$  and  $K$  in net terms and, noting that the resulting coefficient is negative (-.0070), to establish the dynamic stability of the system (cf. Appendix 1).

Consider now the case of a P1 million increase in  $Z$  at year  $t = 0$ , the new level of money supply being sustained thereafter. Assuming that the other exogenous variables remain unchanged, the immediate consequence would be a rise in investment by P.6495 million, in real output by P1.0840 million, in employment by 1.1303 thousand workers, and in the general price level by .0376 index points. Moreover, imports at  $t = 0$  would increase by P.4080 million ( $= .0885 \times 1.0840 + 8.2996 \times .0376$ ), from eq. (4). In the next year ( $t = 1$ ) the economy would have capital stock higher by P.6495 million compared to that where money supply did not increase at  $t = 0$ . Investment would also be higher by P.6450 million ( $= .6495 - .0070 \times .6495$ ), employment by 1.2707 thousand workers ( $= 1.1303 + .2161 \times .6495$ ) and the general price level by 1.2707 index points ( $= 1.1303 + .2161 \times .6495$ ). The higher levels of  $Y$  and  $p$  at  $t = 1$  would mean that imports are also higher by P.4222 million, using eq. (4).

Similar calculations may be made for the succeeding years to generate the year-by-year effects on the endogenous variables as the economy adapts to the P1 million increase in money supply undertaken in the initial year. These

listed for a period of seven years in Table 2.1 of Appendix 2, which also shows in the last row the long-run effect of the exogenous shock. The system is stable so that investment converges to zero, the other variables taking on constant values in the new long-run equilibrium. The deviation of the generated path of each endogenous variable from that "without shock" is seen to approach monotonically over time the long-run deviation. Capital stock would be higher in the new stationary state by ₦92.78 million ( $= .6495/.0070$ ), output by ₦38.81 million ( $= .4066 \times 92.78 + 1.0840$ ), and employment by 21.18 thousand workers ( $= .2161 \times 92.78 + 1.1303$ ). The ₦1 million increase in  $Z$  is also seen to lower the equilibrium value of  $P$  by 0.12 index points and to raise that of  $M$  by ₦2.44 million.<sup>9/</sup>

Tables 2.2 and 2.3 provide similar information relating to once-for-all unit increases in the annual wage and the level of exports, respectively. Very little need be added except to observe that:

(i) The change in the money wage lowers output, employment and investment and raises the price level over time; however, there is a progressive improvement in the trade balance by reducing imports.

(ii) The ₦1 million rise in exports in the initial year leads in the long-run to an increase in output by ₦6.755 million, in employment by 3.590 thousand persons and in imports by ₦.718 million with an accompanying reduction in the general price level by .028 index points. As noted in Section 2 a necessary assumption here is that the money supply is being kept constant by a domestic policy of credit restriction which offsets the increase in  $M$  due to the higher  $X$ .