

Cv

Institute of Economic Development and Research
SCHOOL OF ECONOMICS
University of the Philippines

Discussion Paper No. 68-2

January 19, 1968

**SOME IMPLICATIONS OF A LEXICOGRAPHIC OBJECTIVE
FUNCTION IN DEVELOPMENT PLANNING**

by

José Encarnación, 1928-

NOTE: IEDR Discussion Papers are preliminary versions
circulated privately to elicit critical comment.
References in publications to Discussion Papers
should be cleared with the author.

11/20/68 197
12-29-67

SOME IMPLICATIONS OF A LEXICOGRAPHIC OBJECTIVE FUNCTION IN DEVELOPMENT PLANNING

by

José Encarnación, Jr.

A few years ago Frisch issued "a serious warning regarding a very popular 'planning procedure' [i.e., that] of initially guessing at a 'reasonable' national growth rate that 'could probably be obtained', and from this assumption drawing conclusions regarding the production needed in special sectors of the economy, the size of needed investments, etc. The reason why this method has become so widely used is to be found ... in its simplicity rather than in the fact that it is realistic and rational" [5, p. 1203]. His criticism stems from the methodological point that the target growth rate, as well as other targets relating to other economic variables, should be derived from the country's preference function and the structure of the economy, given the available resources. According to Frisch, there is no assurance that the target growth rate is the optimal one, nor does the procedure yield a determinate result, for "there may be many different development patterns that all give the same rate of growth ... or of some other statistical measure of which one may think" [5, p. 1203].

In this paper we follow Frisch on his methodological point, but we shall argue that a direct determination

of the target values of the relevant variables may be interpreted as itself an approximation of (the parameters of) the preference function. Such an interpretation is made possible on the assumption of lexicographic preferences, which leads in a natural way to specifying a lexicographic objective function as the type of function to optimize in development planning. We then draw some implications in relation to currently used concepts and methods of analysis in this area, with the aim of showing that some unnecessary difficulties are resolved by the explicit assumption of a lexicographic objective function.

I

Since the basically simple concept of a lexicographic preference ordering is as yet relatively unfamiliar, we start with the following brief exposition (see [6][1][3]). Let x, x', x'', \dots denote possible alternatives. Since we are abstracting from uncertainty, an alternative is to be thought of as a state-of-the-world, a complete description of the determinants of choice. Let $U_i(x)$, $i=1,2,\dots$, be real-valued functions such that $U_i(x') > U_i(x'')$ if and only if x' is preferred to x'' on the basis of choice criterion i . (For example, if the rate of unemployment is a choice criterion denoted by k , say, then $U_k(x') > U_k(x'')$ if x' involves a lower unemploy-

ment rate.) We say that x' is preferred to x'' --without qualification as to the basis-- if and only if the first non-vanishing difference in the sequence

$$\min[U_i(x'), U_i^*] - \min[U_i(x''), U_i^*], i=1,2,\dots,$$

is positive, where U_i^* , $i=1,2,\dots$, are certain constants. In other words, the preference ordering of the x 's is given by the lexicographic ordering of the vectors

$$(\min[U_1(x), U_1^*], \min[U_2(x), U_2^*], \dots).$$

Depending on the context, U_i^* may be thought of as indicating a critical minimum level below which is "disaster", a satisfactory or acceptable level, or "satiation" as regards the choice criterion i . In any event, once U_i^* is reached, the criterion i does not influence choice except possibly as a constraint. Thus, suppose that

X_0 = the set of feasible alternatives

$$X_i = \{x : U_i(x) \geq U_i^* \text{ and } x \in X_{i-1}\} \quad i=1,2,\dots$$

Then if X_j is null and X_{j-1} is non-null, the optimal x is that which maximizes U_j subject to $U_i(x) \geq U_i^*$ for all $i < j$.

From a purely formal viewpoint, the above formulation yields the standard "ordinal" utility function by simply putting $U_1^* = \infty$. In this case, U_2, U_3, \dots would

have no roles to play except under unusual circumstances.

The aim of assuming lexicographic preferences, however, is not to generalize the standard real valued utility function but to make the analysis of choice operationally useful. In order to do so, we assume that we can associate the U_i 's with the country's objectives. One can mention the following measurable objectives as typical: to decrease the unemployment level; to increase income per capita or, alternatively, GNP; to promote diversification in the pattern of production (e.g., by expanding the industrial sector relative to the agricultural sector); to increase foreign exchange earnings; to reduce regional inequalities in income distribution; etc. Identifying the U_i 's with such objectives, we may then speak of a lexicographic objective function --or U^* function for brevity-- as the function to optimize in development planning. Accordingly, the optimization problem is that of maximizing some U_j subject to certain constraints on U_i for all $i < j$. Put otherwise, the aim of a development plan would be to advance some specific objective while maintaining other more important (or higher priority) objectives at certain levels of attainment. We may, for example, wish to maximize total saving during the plan-period, provided the following two conditions are satisfied: (i) the number of unemployed at the end of the plan-period does not exceed the number at

the beginning; and (ii) the per capita consumption level does not fall.

A U^* function provides a rationale for Tinbergen's concept of economic policy as the attainment of specific values of the target variables by appropriate choices of the instrument variables in a macro model [12]. His targets correspond to our U_1^* , U_2^* , In the event that not all the targets are attainable, Tinbergen would eliminate one or more of the less important targets as requirements to be satisfied by the solution of the model. It would be entirely consistent with Tinbergen's insight into the nature of economic policy to suppose that he would then maximize the last target variable eliminated --and this is the procedure precisely called for by a U^* function. A U^* function similarly justifies a procedure suggested by Dorfman, who remarks that "the final approach that econometrics suggests to the problem of handling non-comparable benefits is [that] of maximizing performance with respect to some one objective, subject to meeting targets with respect to the other dimensions of performance" [2, p. 199].

II

Economists who are used to the basic assumption that the utility function is real-valued will, of course, say that a U^* function is unreasonable because this does

not allow for substitutability among the objective variables. We are used to consider such questions as: what rate of increase in the price level must one accept in order to reduce the rate of unemployment to a given level (Samuelson-Solow); how much of an increase in GNP is required to reduce the rate of unemployment to a given level (Okun). These are important questions to ask and to answer, but two points are worth observing. First, such questions are raised only after policy-makers have decided on a particular value of an objective variable as acceptable or tolerable. Economists then ask --having been trained to think in terms of trade-offs-- what the attainment of the target would cost in terms of other objective variables. Should it be "too costly" and the policy-makers are so informed, the thinking is that the policy-makers would reconsider their original target and be satisfied with a lower one. The fact remains, however, that policy-makers do think in terms of targets, and this fact should be taken into account in the objective function.

Second, from an operational point of view, it is simpler to operate with a U^* function than with a real-valued function that aims to be realistic. For note that in the case of the latter, rates of substitution among the objective variables are functions of, in general, all the objective variables. If one insists on assuming a real-

valued function

$$U = f[U_1(x), U_2(x), \dots]$$

the difficulties of obtaining f by interviewing the policy-makers (as Frisch suggests) are much greater than in the case of a U^* function. All we need to know are the values U_1^*, U_2^*, \dots which policy-makers in fact talk about. And indeed, the determination of these values, which may be called the parameters of the policymakers' preference function, can be rationalized in terms of simple majority rule (see [3] and [4]).

Thus we conclude that while rates of substitution are obviously relevant in the analysis of the transformation surface relating the objective variables U_1, U_2, \dots , they are relevant for choice only in so far as knowledge of such substitution rates may affect the target values of the objective variables. This does not mean that preference must be represented by an f function rather than by a U^* function. The preference ordering surely depends on what the possibilities are, even though for simplicity, we usually abstract from the set of feasible alternatives in defining the preference ordering. This is true for both the f and U^* functions, and the question of which one to use must be decided by operational usefulness. The clear advantage of a U^* function is that

it provides an immediate rationale for target setting in development planning, which would seem irrational otherwise, as Frisch has claimed.

III

In the current literature on the theory of optimal growth, one considers either a finite horizon or an infinite horizon over which the optimization is to be carried out. (See Koopmans [8] for an admirable survey of some of the issues involved.) Instantaneous utility (or utility during a given period if discrete time is assumed) is made to depend on consumption per capita, and the function maximized is the integral of discounted future utilities. The difficulty that arises with an infinite horizon is that the integral may not converge, so that what is typically done in this case is to make the gratuitous assumption that the rate of discount is sufficiently high in order to assure convergence. (Malinvaud [10] has proposed a definition of "optimal programs" that avoids this problem ; his optimal programs are like Pareto optima, however, and one must be provided with further information to be able to choose among optimal programs.) The difficulty with a finite horizon, on the other hand, is the arbitrary nature of the specific horizon assumed and the amount of "terminal" capital stocks that is required to be left for future generations.

In any event, the typical conclusions that result from models of optimal growth are often intuitively unacceptable. They call for inordinate amounts of saving in the near future in order to build up capital stocks for later years, to the extent that drastic cuts in per capita consumption would have to be imposed. As more than one writer has remarked, this would be intertemporal exchange the wrong way -- later generations would be richer because of present saving, so why should the present poorer generation be made to reduce its already low consumption level? What has gone wrong is the concept of real-valued utility that has been used in these investigations.

(A utility function is nothing but a choice indicator. To assign a higher utility number to a higher consumption level during a given period is harmless: it merely tells us that society would choose a higher to a lower consumption level during that period -- all other things being equal. But what does it mean to speak of an integral of discounted future utilities (or, alternatively, future consumption levels)? Which group -- the present generation (the existing members of the society), a future generation, all generations -- is considered to be expressing a choice? If the concept is to have an operational meaning, we must have to suppose that the utility function is the choice indicator of only the present generation.

But then, can it be seriously assumed that present choices are conditioned by considerations of consumption levels (as such) over the next hundred years or thousand years? It seems not. There are different categories of needs, and present needs dominate future needs belonging to the same category. It will not do to consider the consumption level at some remote time and neglect the values at the present time of other objective variables.

What may be done with a U^* function is to assign dates to its arguments in such a way that generally speaking, objective variables in the nearer future occur "earlier" in the vector $[U_1(x), U_2(x), \dots]$. Suppose, for example, that we are considering only the three objective variables employment, consumption and saving, in that order. Then U_1 refers to employment, U_2 to consumption, and U_3 to saving all in year 0; after which U_4 refers to employment, U_5 to consumption, and U_6 to saving in year 1; and in general U_{3t+i} refers to the i th objective variable at year t . Any number of variables can be handled in this way, and optimization of the U^* function will yield a determinate solution. Note that there is no need here to use a rate of time discount.

Discount rates are widely used in the literature (cf. [9]) for the purpose of evaluating a stream of future events (consumption, income, costs, benefits, "utilities",

...times without a proper appreciation of their ... validity. A discount rate may be appropriate in ...-making of an individual or a business firm if ... operates in a market where it can trade present goods for future goods and vice versa. This would be on the assumption that the individual's preferences over possible time paths are representable by a numerical function, in which case the optimization problem is similar to that of the standard theory of consumer behavior. One considers trade-offs, at the margin, between present goods and future goods. But note here, first, that the discount rate is a function of the quantities of present goods and future goods; hence there can hardly be any such thing as the appropriate [↑] rate of discount. Second, the concept is inapplicable when constraints are just satisfied at the optimum. For instance, if a minimum consumption constraint at the present time must be satisfied, it makes no sense to compare alternative streams of consumption by means of a discount rate and then conclude that present consumption should be reduced further. At the macro level, considering that consumption per capita is typically low in a developing economy, we may expect that a minimum constraint on consumption is indeed required.

As we have already seen, a U^* function makes possible the consideration of time without having to use a rate of discount. We have discussed the point at the macro

level. The next question is whether the same would hold true at the micro level, specifically in the evaluation of possible investment projects.

According to current theory (cf. [11]), investment projects are to be ranked according to their present values of total benefits less total costs; if there is a constraint on available investment funds, higher ranking projects are selected until the available funds are exhausted. There is an extensive literature on the appropriate rate of discount, since a discount rate is considered necessary in order to evaluate time streams (cf. [7]). This is so, however, only because of the unnecessary restriction of the analysis of this matter to real-valued indicators of worth. Once we accept the fact that a development plan aims to accomplish a multiplicity of objectives, and therefore investment projects should be evaluated in terms of those objectives, reliance on discount rates is no longer necessary. For, suppose that

$$U_{ik} = F_{ik}(y_1, \dots, y_n) \quad \begin{array}{l} k = 1, 2, \dots, n \\ i = 1, 2, \dots \end{array}$$

where U_{ik} is the contribution made by sector or project k towards the i th objective, and y_k is the amount of investment funds allocated to the k th sector. Under a lexicographic ordering, our problem would be to maximize

$$U_j = \sum_k U_{jk}$$

subject to

$$\sum_k U_{ik} \geq U_i^* \quad (i=1,2,\dots, j-1)$$

$$\sum_k y_k \leq Y$$

where Y is the budget constraint. There are three possibilities: (i) There is no solution, in which case we replace U_j by U_{j-1} in the problem; (ii) The solution gives $U_j \geq U_j^*$, and so we replace U_j by U_{j+1} in the problem; (iii) The solution gives $U_j < U_j^*$, in which case we have the optimal allocation.

The F_{ik} may be difficult to determine precisely in practice, but the point is that in principle, rates of time discount are not necessary. The method of computing present values has its own difficulties, even if time discounting were meaningful. Future costs and benefits are uncertain, and relative prices would obviously change as a result of projects undertaken.

On the other hand, in the problem as we have posed it, since present objectives have priority over later objectives, precision as to future consequences is of lesser importance. Information demands would be less exacting and the results would be less sensitive to varying estimates

of the distant consequences of decisions undertaken. This is because the optimal allocation is likely to be determined by constraints and objectives which, although they may have long-term implications, are expressed in terms of categories that are nearer to the present. (Thus for example, if saving is the objective to be maximized, one does not need to guess at the future course of prices, etc.) Moreover, and more important, even the nearer consequences of decisions need not all be known with any great exactness. We do not care to what extent the more important objectives are more than satisfied --the constraints in the problem are inequalities-- and hence there is much more room in which to accommodate imprecision in our data and our knowledge of the causal relationships involved.

IV

We have discussed several points which suggest distinct advantages in pursuing a lexicographic objective function approach to development planning, as compared to the standard real-valued function approach.

(i) Rates of substitution are extremely difficult to determine, even when only one individual's views --the President's or the Prime Minister's-- are consulted. When we have to consider the views of a group, say those of the

Cabinet or the Congress, the problem has no solution even in principle (except in the most trivial cases). In contrast, in the case of a U^* function, we need only ask what levels of the objectives are acceptable, and which objectives would one be willing to sacrifice if not all of them can be attained. Moreover, the group decision problem has a solution (see [4]).

(ii) The concept of a social rate of time discount, which is premised on real-valued social preference functions, compounds further the difficulties present in the standard approach. In contrast, a U^* function takes account of the difference between the present and the future --not by giving them different weights and then supposing that they are made comparable thereby, a procedure which is misleading at best-- but by locating them as different elements in a vectorial representation of choice.

(iii) The requirements in regard to the accuracy of data and the exactness of our knowledge of causal relations are less stringent if a U^* function is to be optimized. The inequality constraints pertaining to the more important objectives in a U^* function give room for error, whereas in the standard formulation, the value of the objective function depends on the exact values of all the variables.

(iv) Justification of targets in development planning is straightforward in terms of a U^* function, whereas on the assumption of a real-valued objective function, any such justification would be strained. A theoretical framework is thus provided for actual practice in development planning, which would otherwise be subject to Frisch's structures.

What is suggested by all this is the potential fruitfulness of research along lines different from the current ones. It seems pointless to formulate a model of optimal economic growth, deduce the nature of its solution, and find that development planners and societies are "irrational" only because of the oversimplification that social choice is representable by a numerical function. The more appropriate task of the economist is to determine the objectives in a development plan, and then show how the more important objectives may be attained.

R E F E R E N C E S

- [1] Chipman, J.S. "The Foundations of Utility," *Econometrica*, vol. 28, April 1960, pp. 193-224.
- [2] Dorfman, R. "Econometric Analysis for Assessing the Efficacy of Public Investment," in [13], pp. 187-205. ✓
- [3] Encarnación, J. "Optimum Saving and the Social Choice Function," *Oxford Economic Papers*, vol. 16, July 1964, pp. 213-220.
- [4] Encarnación, J. "A Model of Group Decision-Making," (abstract) *Management Science*, vol. 13, June 1967, p. C-338.
- [5] Frisch, R. "Selection and Implementation: The Econometrics of the Future," in [13], pp. 1197-1204.
- [6] Georgescu-Roegen, N. "Choice, Expectations and Measurability," *Quarterly Journal of Economics*, vol. 68, November 1954, pp. 503-534.
- [7] Harberger, A.C. "Techniques of Project Appraisal," in *National Economic Planning*, ed. M.F. Millikan, New York: NBER, 1967, pp. 131-149.
- [8] Koopmans, T.C. "On the Concept of Optimal Economic Growth," in [13], pp. 225-287.
- [9] Malinvaud, E. "Interest Rates in the Allocation of Resources," in *The Theory of Interest Rates*, ed. F.H. Hahn and F.P.R. Brechling, London: Macmillan, 1965, pp. 209-241.
- [10] Malinvaud, E. "Croissances optimales dans un modèle macroéconomique," in [13], pp. 301-378.
- [11] Prest, A.R. and Turvey, R. "Cost-Benefit Analysis: A Survey," *Economic Journal*, vol. 75, December 1965, pp. 683-735.
- [12] Tinbergen, J. *Economic Policy: Principles and Design*, Amsterdam: North-Holland, 1956.
- [13] *Study Week on the Econometric Approach to Development Planning* (Vatican City, 1963), Amsterdam: North-Holland, 1965.