Given the above information, it can be inferred at least that there appears to be no strong difference between the two measures of output in the production function.

How do these estimates differ from those derived from regressions involving observations from fixed asset sizes per industry group? Figure 4.3 shows the scatter of these estimates, whenever paired observations are available. The presence of nonsignificant estimates in one or another set of estimates has reduced the number of possible paired observations.

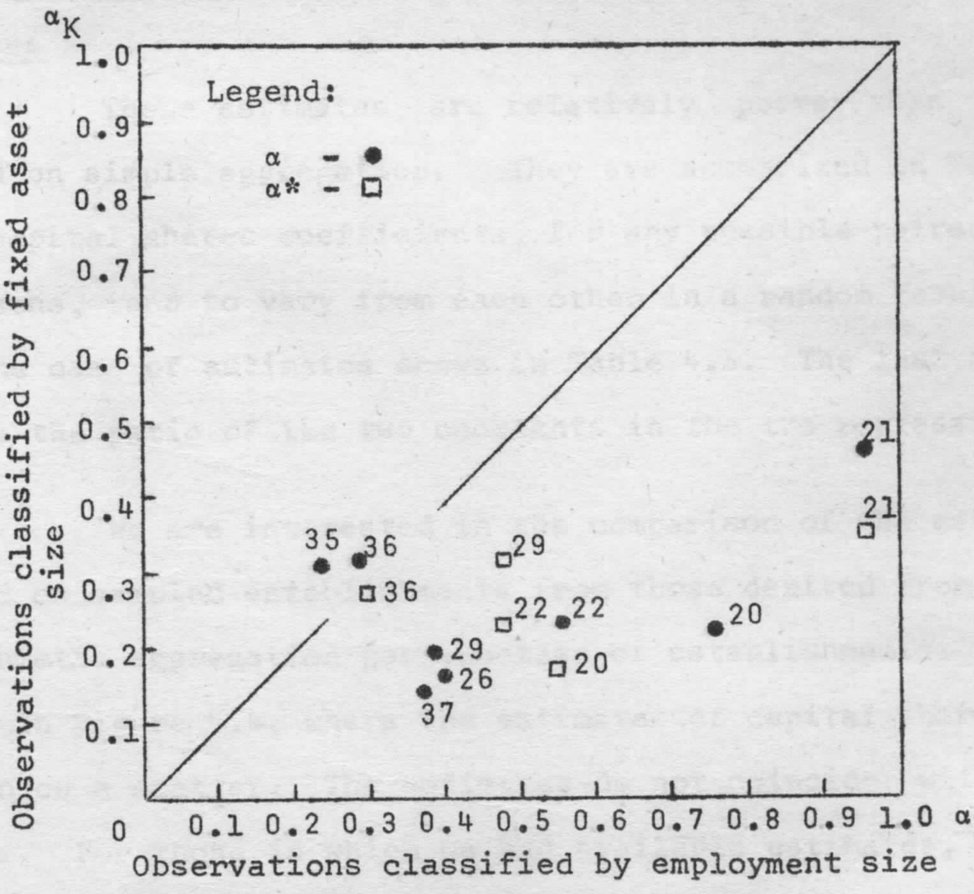

Figure 4.3. Comparison of Estimates of Capital Shares of Regressions Based on Different Data Classification

It appears that the estimates of Cobb-Douglas production functions utilizing observations divided by employment size tend to yield higher estimates for the share of capital compared to estimates based on fixed assets classification. We defer to later pages to explain this phenomenon. Intuitively, if the method of classifying observations do not create any specific bias on results, the two estimates would be equivalent.

Estimates Based on Sampling of Microunits of Employment Classes

These estimates are relatively poorer than those based on simple aggregation. They are summarized in Table 4.6. The capital shares coefficients, for any possible paired comparisons, tend to vary from each other in a random fashion, as in the case of estimates shown in Table 4.5. The last column shows the ratio of the two constants in the two regressions.

We are interested in the comparison of the estimates based on sampled establishments from those derived from sample arithmetic aggregation per subclass of establishments. This is done in Figure 4.4, where the estimates of capital shares are drawn on a scatter. The estimates do not coincide with each other. For those in which we had available estimates, except for beverages (ISIC 21) whose capital shares from aggregated

Table 4.6. CAPITAL SHARES, COMPARED FOR PRODUCTION FUNCTIONS BASED ON GROSS SALES AND VALUE ADDED

Data Observations Sampled

ISIC Code	Industry	α_K^* Based on Gross Sales	α_K Based on Value Added	$\alpha_K-\alpha_K^*$	Ratio of Constants V/G
20	Manufactured Food	0.333 (0.079)	0.453 (0.082)	0.120	0.332
21	Beverages	0.359 (0.151)	0.282 (0.220)	-0.077	0.566
22	Tobacco	n.s.	n.s.		0.375
23	Textiles	n.s.	0.266 (0.202)		0.283
24	Footwear & apparel	n.s.	n.s.		0.386
25	Wood & cork	0.362 (0.144)	0.377 (0.188)	0.015	0.447
26	Furniture & fixtures	0.643 (0.184)	0.529 (0.186)	-0.114	0.612
27	Paper products	n.s.	n.s.		0.273
28	Printed & published materials	n.s.	n.s.		0.610
29	Leather products	0.346 (0.154)			0.422
30	Rubber products	0.274 (0.218)	0.324 (0.232)	0.050	0.366
31	Chemical products	n.s.	n.s.		0.380
33	Non-metallic mineral	n.s.	n.s.		0.483
34	Basic metal	n.s.	n.s.		0.423
35	Metal products	0.447 (0.102)	0.406 (0.123)	-0.041	0.432
36	Machinery, non-electric	n.s.	n.s.		0.665
37	Electrical machinery	0.471 (0.169)	0.449 (0.125)	-0.022	0.539
38	Transportation	n.s.	n.s.		0.495

n.s. - not significant

Standard errors of coefficients in parentheses.

Ratio of constants is ratio of estimated intercepts for value added to gross sales regressions.

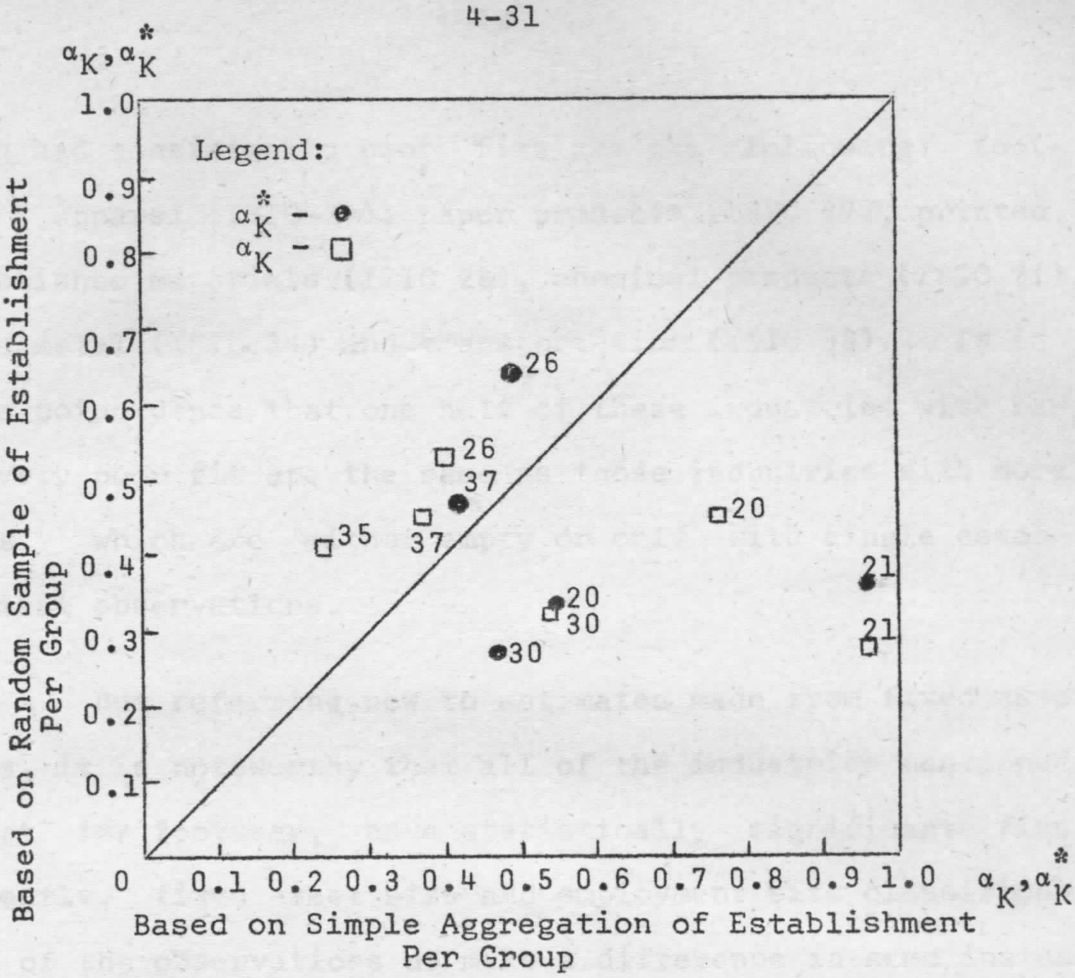

Figure 4.4. Capital Shares Estimates Using Establishment Employment Size Observations

data seem to be quite high relative to capital shares based on sampled data, the number of times the estimates on aggregated data exceeded those based on sampled data was about the same as when they fell short of them.

Industries with Poor Production Function Fits

It is interesting to note how the production function fits vary depending on the observations used. In many cases, the same industries did not yield good fits for the two types of employment size observations. The industries which had consistently poor fits are the following: footwear & apparel (ISIC 24), paper products (ISIC 27), printed & published materials (ISIC 28), chemical products (ISIC 31), basic metal (ISIC 34) and transportation (ISIC 38). It is not a coincidence that one half of these industries with relatively poor fit are the same as those industries with more cells which are either empty or only with single establishment observations.

But referring now to estimates made from fixed asset sizes, it is noteworthy that all of the industries mentioned, except for footwear, have statistically significant fits. Evidently, fixed asset size and employment size classification of the observations do make a difference in some instances. Since capital-labor ratios differ from among different establishments within the same industry, at least in those industries where such differences are marked in their establishment samples, the shift in subgroup locations of certain establishments cause changes in the error composition of the regression models utilized.

Needless to say, we can analyze also the fixed asset regressions and discover certain industries in which their poor fits improved with the utilization of employment sizes as regression data. This is most apparent from the fits derived for textiles (ISIC 23), rubber products (ISIC 30), non-metallic mineral products (ISIC 33), and electrical machinery.

Three-Factor Production Functions

The introduction of a third factor helps to clarify an interesting aspect of production. The production of an output is the result not only of the use of the primary -- inputs labor and capital -- but also of intermediate inputs which are purchased from other production units. While homogeneous inputs of labor and capital are assumed to be used, in addition there is also an aggregate homogeneous intermediate input, J, without which some production activity will not be possible. Even in an enterprise where everything which is produced is only the combination of labor and capital, there is often the need to use raw materials, which have to be transformed into an output. In many instances, this third input is ignored.

Thus, Evsey Domar (1966) recently complained:[5]

> I wonder what has happened ... to material inputs? If they are omitted because of the lack of required data, we have an answer, even if, to my mind, a regrettable one. But usually an author begins his paper with the model that he would like to fit; then he apologizes for the lack of data

[5]Domar (1966) in Brown (ed.) (1966), p. 471-2.

> and fits a different one. I have not found any apologies for omitting material inputs from both sides of the equation and thus working with value added on the one side and with only labor and capital on the other. Is this then the desired method? And yet it seems to me that a production function is supposed to explain a productive process, such as the making of potato chips from potatoes (and other ingredients), labor and capital. It must take some ingenuity to make potato chips without potatoes....

Referring back to Chapter 2, the production function with three factors is written as

$Q = A L^{1-\beta-\gamma} K^{\beta} J^{\gamma}$

$$Q = f(K, L, J)$$

$$= L\ f(K/L), J/L, 1),$$

$\frac{Q}{L} = A\left(\frac{K}{L}\right)^{\beta}\left(\frac{J}{L}\right)^{\gamma}$

since we assume constant returns to scale. The Cobb-Douglas production function 2-digit industry is given by

$$Q = A\ K^{\alpha_K}\ L^{\alpha_L}\ J^{\alpha_J}$$

where $\alpha_K + \alpha_L + \alpha_J = 1$. Through the use of the estimation procedure given by equation (2.7a) in Chapter 2, direct estimates are made for α_K and $\ddot{\alpha}_J$. Thus, since three inputs are used in production, the distribution of output becomes a three-way division.

We have used two different measures of output -- gross sales and value added -- in estimating the respective try production functions. This is analogous to the effects of the rence in the measures of output on the estimated production functions. lue added and gross sales are related in any special way, then the luction of a third input in the production function will be valid, ever measure of output is used. Whether or not the introduction of ediate inputs purchased from other production units will improve the ation yielded by a two-factor production function is of course an- issue. We shall turn to this towards the end of this chapter.

Estimates. Tables 4.7 and 4.8 summarize the results of the estimates of production functions, using two output concepts and two types of observations for the regressions -- a total of four sets of estimates. The two observations used reported for any of the four possible estimation procedures. In three other cases, only one set of estimates are reported.

To examine briefly the difference between coefficients of capital estimates based on the different regressions by employment size, we present Figure 4.5. Of the 9 pairs of estimates available, five have capital shares coefficients based on sampled data per employment class which are higher than those based on simple aggregation. Four others have the reverse inequality. In general, examination of the statistical quality of the estimates by reference to the standard errors relative to the estimates, shows that those based on simple aggregation are better.

Table 4.7. THREE-FACTOR COBB-DOUGLAS PRODUCTION FUNCTIONS
Factor Shares Based on Aggregated Data

ISIC Code	Industry	Gross Sales			Value Added		
		α_K^*	α_J^*	α_L^*	α_K	α_J	α_L
20	Manufactured Food	0.393 (0.078)	0.367 (0.097)	0.240	*	*	*
21	Beverages	0.730 (0.256)	0.282 (0.254)	*	*	*	*
22	Tobacco	0.264 (0.091)	0.668 (0.149)	0.068	0.453 (0.156)	0.348 (0.256)	0.19
23	Textiles	*	*	*	*	*	*
24	Footwear & apparel	0.244 (0.066)	0.396 (0.022)	0.360	0.238 (0.161)	0.078 (0.053)	0.68
25	Wood & cork	*	*	*	*	*	*
26	Furniture & fixtures	*	*	*	*	*	*
27	Paper products	*	*	*	*	*	*
28	Printing	*	*	*	*	*	*
29	Leather products	*	*	*	*	*	*
30	Rubber products	0.068 (0.052)	0.985 (0.063)	*	0.147 (0.126)	0.973 (0.152)	*
31	Chemical products	*	*	*	*	*	*
33	Non-metallic mineral	0.184 (0.060)	0.864 (0.100)		0.326 (0.113)	0.745 (0.188)	
34	Basic metal	*	*	*	*	*	*
35	Metal products	0.093 (0.048)	0.721 (0.041)	0.186	0.205 (0.108)	0.413 (0.093)	0.3
36	Machinery, non-electric	0.166 (0.160)	0.359 (0.117)	0.475	0.222 (0.213)	0.190 (0.156)	0.5
37	Electrical machinery	0.121 (0.057)	0.545 (0.068)	0.334	0.264 (0.123)	0.191 (0.146)	0.5
38	Transportation	*	*	*	*	*	*

*Not recorded, because at least one factor has statistically not signi cant coefficient. This notation should not be confused with th superscripts of the gross sales factor shares estimates.

Table 4.8. THREE-FACTOR COBB-DOUGLAS PRODUCTION FUNCTIONS
Factor Shares Based on Sampled Data

ISIC Code	Industry	Gross Sales			Value Added		
		α_K*	α_J*	α_L*	α_K	α_J	α_L
20	Manufactured Food	0.142 (0.040)	0.755 (0.755)	0.102	0.358 (0.094)	0.377 (0.214)	0.26
21	Beverages	*	*	*	*	*	*
22	Tobacco	0.128 (0.088)	0.782 (0.097)	0.090	*	*	*
23	Textiles	*	*	*	0.212 (0.176)	0.457 (0.189)	0.33
24	Footwear & apparel	0.106 (0.091)	0.489 (0.075)	0.405	*	*	*
25	Wood & cork	*	*	*	*	*	*
26	Furniture & fixtures	0.116 (0.134)	0.567 (0.106)	0.317	*	*	*
27	Paper products	*	*	*	*	*	*
28	Printing	*	*	*	*	*	*
29	Leather products	*	*	*	*	*	*
30	Rubber products	0.179 (0.128)	0.579 (0.127)	0.242	0.263 (0.213)	0.380 (0.211)	0.53
31	Chemical products	0.092 (0.075)	0.722 (0.072)	0.186	*	*	*
33	Non-metallic mineral	0.040 (0.038)	0.893 (0.086)	0.067	0.095 (0.084)	0.835 (0.189)	0.07
34	Basic metal	*	*	*	*	*	*
35	Metal products	0.152 (0.081)	0.648 (0.127)	0.200	*	*	*
36	Machinery, nonelec.	*	*	*	*	*	*
37	Electric machinery	0.133 (0.069)	0.573 (0.059)	0.294	0.288 (0.120)	0.274 (0.104)	0.43
38	Transportation	*	*	*	*	*	*

*Not recorded, because at least one factor has statistically not significant coefficient. This notation should not be confused with the superscripts of the gross sales factor shares estimates.

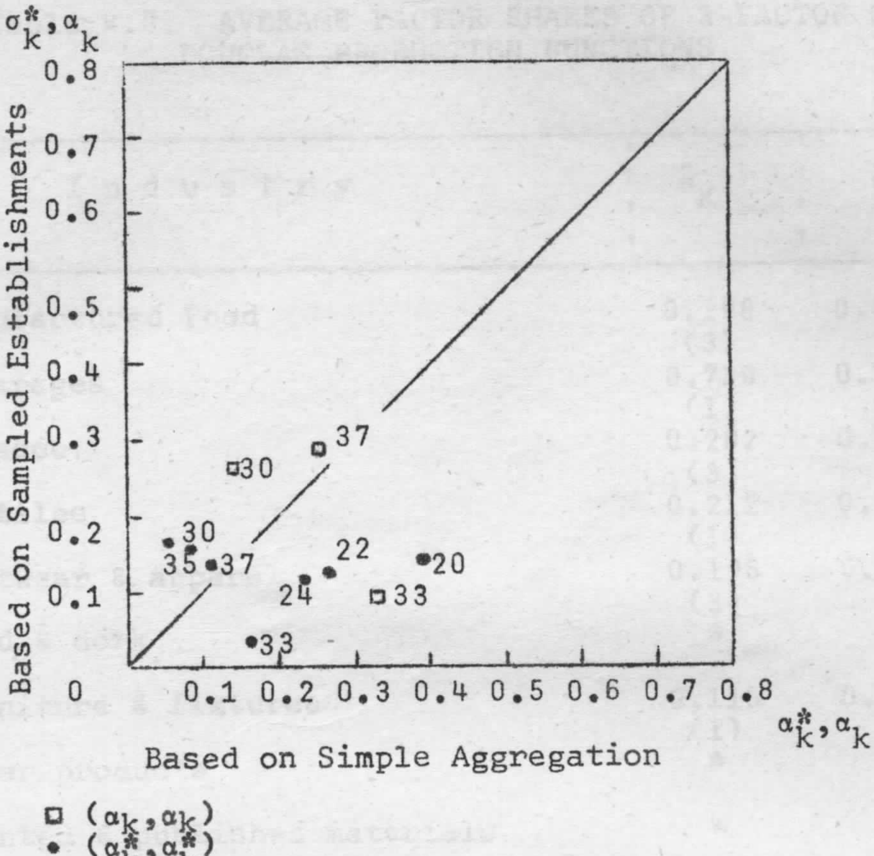

Figure 4.5. Capital Shares Estimates
from Three-Factor Cobb-Douglas Production Functions

Although we recognize that the said scatter is not sufficient to justify the conclusion that the estimates for both types of data observations are in general equal, it may be useful to average the values of the factor shares estimates. This has the advantage of not only compressing already enormous information, but it also adds to a simplification of the analysis.

Thus, in Table 4.9, average values of the factor shares coefficients, which we now denote $\hat{\alpha}_K$, $\hat{\alpha}_J$, and $\hat{\alpha}_L$, are reported. The total number of the coefficient estimates

Table 4.9. AVERAGE FACTOR SHARES OF 3-FACTOR COBB-DOUGLAS PRODUCTION FUNCTIONS

ISIC Code	Industry	$\hat{\alpha}_K$	$\hat{\alpha}_J$	$\hat{\alpha}_L$
20	Manufactured Food	0.298 (3)	0.500	0.202
21	Beverages	0.730 (1)	0.282	*
22	Tobacco	0.282 (3)	0.599	0.119
23	Textiles	0.212 (1)	0.457	0.331
24	Footwear & apparel	0.196 (3)	0.321	0.483
25	Wood & cork	*	*	*
26	Furniture & fixtures	0.116 (1)	0.567	0.317
27	Paper products	*	*	*
28	Printed & published materials	*	*	*
29	Leather products	*	*	*
30	Rubber products	0.164 (4)	0.729	0.107
31	Chemical products	0.092 (1)	0.722	0.186
33	Non-metallic mineral	0.161 (4)	0.834	0.005
34	Basic metal	*	*	*
35	Metal products	0.150 (3)	0.594	0.256
36	Machinery, non-electric	0.194 (2)	0.274	0.532
37	Electrical machinery	0.202 (4)	0.396	0.402
38	Transportation	*	*	*

Number in parentheses is number of estimates from which average is derived taken from previous column.

which have been averaged are reported in parentheses underneath the average capital share. As in the earlier estimation procedure, we derived the $\hat{\alpha}_L$ indirectly after $\hat{\alpha}_K$ and $\hat{\alpha}_J$ are known. Do the figures for $\hat{\alpha}_K$ and $\hat{\alpha}_L$ compare with those derived from two factor production functions? To make these comparable, we weight the $\hat{\alpha}_K$ and $\hat{\alpha}_L$.

Let $$\hat{\hat{\alpha}}_K = \frac{\hat{\alpha}_K}{\hat{\alpha}_K + \hat{\alpha}_L}$$

and $$\hat{\hat{\alpha}}_L = \frac{\hat{\alpha}_L}{\hat{\alpha}_K + \hat{\alpha}_L}.$$

These numbers are now comparable to those we have estimated for the two factor production functions.

In order to ~~compared~~ compare the two-factor production function capital shares with the adjusted capital shares derived from the 3-factor production functions, some simplifications are performed. Recall that, in general, the estimates of capital shares appeared not to be seriously different whether the measure of output is gross sales or value added. In view of this, for every pair of estimates, α_K (based on value added) and α_K^* (based on gross sales) which are nonnegative, it may be possible to take their simple average, without drastically affecting our results. In addition, for those regressions in which only either one of α_K or α_K^* are significant, only the estimate of the significant capital share coefficient is reported. This happened in three cases for estimates based on simple aggregation and once on sampled observations. In order to identify fully these new estimates, we spell the out as follows:

$\bar{\alpha}_K$ = "average" estimates for α_K based on simple aggregation

$\bar{\alpha}_K'$ = "average" estimate for α_K based on "sampled" observations per employment class.

Table 4.10 shows the estimates based on two-factor and three-factor production functions, with the capital shares estimates aggregated for the two-factor production functions and adjusted for the 3-factor. It is interesting to note that the capital shares estimates in the three-factor production functions appear to diverge significantly from the two-factor estimates, as can be seen from the last two columns. There are 9 industries for which at least one paired comparison of the capital shares was possible. Of these, it can be said that in six cases, the differences of the capital share estimates were not too important, whether the estimates are by

Table 4.10. CAPITAL SHARES FROM TWO-FACTOR AND THREE-FACTOR PRODUCTION FUNCTIONS

ISIC Code	Industry	Two-Factor α_K		$\hat{\hat{\alpha}}_K$	$\bar{\alpha}_K-\hat{\hat{\alpha}}_K$	$\bar{\alpha}'_K-\hat{\hat{\alpha}}_K$
		$\bar{\alpha}_K$	$\bar{\alpha}'_K$			
20	Manufactured Food	0.655	0.393	0.596	0.059	-0.203
21	Beverages	0.963	0.320	*	*	*
22	Tobacco	0.524	*	0.705	-0.180	*
23	Textiles	0.150[a]	0.266[a]	0.390	-0.240	-0.124
24	Footwear & apparel	0.257	*	0.289	-0.032	*
25	Wood & cork	*	0.370	*	*	*
26	Furniture & fixtures	0.448	0.586	0.268	0.180	0.318
27	Paper products	*	*	*	*	*
28	Printing	*	*	*	*	*
29	Leather products	0.436	0.276	*	*	*
30	Rubber products	0.505	0.299	0.442	0.063	0.143
31	Chemical products	*	*	0.331	*	*
33	Non-metallic mineral	*	*	*	*	*
34	Basic metal	*	*	*	*	*
35	Metal products	0.240[a]	0.426	0.369	-0.129	0.057
36	Machinery, non-electric	0.300	*	0.267	0.033	*
37	Electric machinery	0.390	0.460	0.334	0.056	0.126
38	Transportation	*	*	*	*	*

[a]These are based on only one significant capital share estimates.

two-factor production functions or 3-factor. These industries are manufactured food (ISIC 20), footwear & apparel (ISIC 24), rubber products (ISIC 30), metal products (ISIC 35), nonelectric machinery (ISIC 36) and electric machinery (ISIC 37). The rest showed relatively high differences in the capital shares estimates. The other three industries had relatively high differences in their capital shares coefficients for a number of reasons. One reason which explains the high capital share (and relatively the low value of labor) of tobacco is that the entry of a third input <u>J</u> tended to cause all the variations to be explained by the new variable. Thus, capital, although still significant, gets a relatively minor share of the explanation of the regressions. The other reason is that the capital shares estimates are based only on a single estimate rather than on several; the other regressions for the industry from which these estimates are taken contain estimates which are not significantly different from zero. Textiles (ISIC 23) is affected by the poor fit of the two-factor production functions estimates and furniture & fixtures (ISIC 26) by the poor fits of the other 3-factor production function estimates.

The above summarizes the estimates of Cobb-Douglas production functions, comparing both the two- and three-factor production functions. When adjusted, the 3-factor estimates had capital coefficients which were in general not too different from the corresponding factor shares derived from two-factor production functions. But as we have pointed out, the three-factor production functions did show relatively poorer fits than the two-factor cases. The addition of intermediate inputs, J, often had the effect of swamping the influence of capital or of labor in explaining the variation of output in the regression estimates. This tendency is apparently not alone due to statistical factors, e.g., collinearity between the observations of the inputs or an interdependence of error components in the measurements of each input. It shows the relative importance of raw materials to the production relation, which is often neglected in analysis of production.

Having established, however imperfectly, that the capital shares (and consequently, the labor shares) of the production functions, whether two-factor or three-factor are not too far from each other in value, it is possible for us to go a little further in the analysis of production in Philippine manufacturing industries.

Best Cobb-Douglas Factor Shares. A necessary step that should be met before further analysis is possible is to settle on the best factor shares per industry, in view of the various estimates of Cobb-Douglas production functions reported in this chapter. The objective criterion useful in this regard is to examine the statistical properties of the alternative estimates of capital shares. This was done by comparing the values of the t-statistics for each set of estimates, choosing that one with the highest t-value.[6] Thus, we choose the coefficient whose error term relative to the estimated coefficient is smallest. The choice was initially confined to the estimates based on employment size regressions, considering what had been said earlier. Estimates based on simple aggregation of employment sizes appeared to be superior when compared directly with those based on sampled establishments. One half of the best capital shares estimates from all the industry group regressions are based on simple aggregation of employment sizes. Four capital share estimates are based on sampled establishments. The other estimates are based on fixed asset grouping of establishments. These are the industries whose capital shares estimates did not appear significant in any of the employment size production function regressions.

We show these estimates in Table 4.11, taking care to note their specific origins. The estimates shown in this table will now be the basis of succeeding comments on factor shares in Philippine manufacturing.

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[6]The t-statistic is easily derived from the coefficient estimate, α_K, and the standard error, since $t = \alpha_K/\text{standard error}$.

Table 4.11. STATISTICALLY BEST COBB-DOUGLAS CAPITAL SHARES

ISIC Code	Industry	From Table 4.5		From Table 4.6		From Table 4.3		$\bar{R}^2$
		α_K Value Added	α_K* Gross Sales	α_K Value Added	α_K* Gross Sales	α_K Value Added	α_K* Gross Sales	
20	Manufactured Food	*	0.545 (5.924)	*	*	*	*	0.69
21	Beverages	*	0.963 (6.463)	*	*	*	*	0.73
22	Tobacco	0.566 (4.131)	*	*	*	*	*	0.54
23	Textiles	*	*	0.266 (1.317)	*	*	*	0.05
24	Footwear & apparel	0.257 (1.521)	*	*	*	*	*	0.10
25	Wood & cork	*	*	*	0.362 (2.514)	*	*	0.27
26	Furniture & fixtures	*	*	*	0.643 (3.494)	*	*	0.53
27	Paper products	*	*	*	*	*	0.258 (3.739)	0.66
28	Printing	*	*	*	*	0.247 (2.839)	*	0.50
29	Leather products	*	0.481 (3.943)	*	*	*	*	0.68
30	Rubber products	0.542 (2.221)	*	*	*	*	*	0.26
31	Chemical products	*	*	*	*	0.296 (3.253)	*	0.64
33	Non-metallic mineral	0.520 (3.421)	*	*	*	*	*	0.45
34	Basic metal	*	*	*	*	0.300 (3.947)	*	0.69
35	Metal products	*	*	*	0.447 (4.382)	*	*	0.59
36	Machinery, nonelec.	*	0.305 (1.466)	*	*	*	*	0.10
37	Electric machinery	0.367 (3.784)	*	*	*	*	*	0.49
38	Transportation	*	*	*	*	*	0.226 (2.568)	0.45

Note: Unlike the other tables, the numbers in parentheses are t-statistics, not coefficient standard errors.

$\bar{R}^2 = R^2$, corrected for degrees of freedom.

Enfin!

Actual and Estimated Factor Shares. Aside from posing the question of the distributional implications of estimates of Cobb-Douglas factor shares, there are key issues in production function analysis such as allocation of resources and the explanation of growth of productivity. Since we have only one cross-section from which these estimates are based, it is not possible to make any specific statements about growth of productivity. However, the factor shares estimates are relevant if, in discussions of productivity growth, they are assumed as given and the production function assumed incorporates productivity changes which are neutral with respect to the marginal rates of substitution of the factors. This is the now well-known disembodied technical progress incorporated into a Cobb-Douglas production function, as introduced by Solow (1957). We shall not get into this discussion here,[7] but we shall attempt to answer the relation of resource allocation with the factor shares estimates.

It is obvious from Table 4.12 that the actual share of capital (in the sense of non-labor shares) exceed the estimated capital shares for almost all industries, with the exception of beverages and furniture & fixtures. Theoretically, if there were no wide variations in observations of output and the inputs,

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[7]However, see J.G. Williamson and G.P. Sicat (1968).

when?
1960?

Table 4.12. ACTUAL COMPARED TO STATISTICALLY BEST FACTOR SHARES ESTIMATES

ISIC Code	Industry	Actual		Statistically	
		r Non-Labor Share	w Labor Share	Best α_K	Best α_L
20	Manufactured Food	0.81	0.19	0.545	0.455
21	Beverages	0.79	0.21	0.963	0.037
22	Tobacco	0.76	0.24	0.566	0.434
23	Textiles	0.54	0.46	0.266	0.734
24	Footwear & apparel	0.51	0.49	0.257	0.743
25	Wood & cork	0.51	0.49	0.362	0.638
26	Furniture & fixtures	0.51	0.49	0.643	0.357
27	Paper products	0.72	0.28	0.258	0.742
28	Printed & published materials	0.46	0.54	0.247	0.753
29	Leather products	0.57	0.43	0.481	0.519
30	Rubber products	0.74	0.26	0.542	0.458
31	Chemical products	0.77	0.23	0.296	0.704
33	Non-metallic mineral	0.71	0.29	0.520	0.480
34	Basic metal	0.63	0.37	0.300	0.700
35	Metal products	0.66	0.34	0.447	0.553
36	Machinery, non-electric	0.66	0.34	0.305	0.695
37	Electrical machinery	0.67	0.33	0.367	0.633
38	Transportation	0.55	0.45	0.226	0.774
	Total Manufacturing	0.73	0.27		

if the production function is Cobb-Douglas and there is competitive factor pricing, we would expect that the final division of output is to be in accordance with the marginal productivities of the respective factors. The following equation for each industry has to be satisfied,

$$\frac{R_i\ K_i}{W_i\ L_i} = \frac{(\partial Q/\partial K)K_i}{(\partial Q/\partial L)L_i} = \frac{\alpha K_i}{\alpha L_i}$$

or, simply (removing the i notation for each industry)

$$\frac{R}{W} = \frac{\partial Q/\partial K}{\partial Q/\partial L}$$

Q is in pe valu

$$\neq \frac{\alpha_K}{\alpha_L}$$

where R is the gross rental on capital per unit per year, W the wage rate, the respective marginal products given by the partial derivatives, and α_K and α_L as already defined. Since output, Q, is exhausted by its allocation into either a return to capital or to labor, we have

$$Q = R + W.$$

$Q = RK + WL$

$1 = \frac{RK}{Q} + \frac{WL}{Q}$

Dividing both sides by Q, we have