$$1 = R/Q + W/Q$$

or $$1 = r + w$$

where r and w are recorded fractions of output shared by capital and labor respectively. Since $(R/W) = (r/w)$, we have the following equation

$$\frac{r}{w} = \frac{\alpha_K}{\alpha_L} .$$

As can be seen from Table 4.12, however, the inequality

$$\frac{r}{w} > \frac{\alpha_K}{\alpha_L}$$

holds, suggesting a discrepancy between the actual division of output between the two inputs and the factor shares implied by the production function. The following equality is therefore suggested,

$$\lambda(r/w) = \alpha_K/\alpha_L$$

or

(4.14) $$\lambda = (\alpha_K/\alpha_L)/(r/w)$$

where $\lambda<1$, as suggested by the inequality. λ is a factor that corrects for the actual share of capital, r, as a ratio to the labor share when the statistical estimates of the factor shares are assumed to be adequate. The values of λ are given in Table 4.13.

This deviation of the recorded factor rewards from their marginal physical productivities, as the inequality in their ratios show, may be partly interpreted as the deviation of an "ideal" situation from the actual. It is quite obvious that marginal productivity factor pricing is a hard condition to meet since factor markets are full of imperfections, especially in economies like the Philippines. An example concerns the capital market. Its imperfection is obvious from the evidence of a relatively small stock exchange and money markets. Moreover, capital funds held by governmental institutions are rationed in a way. In addition, there exist a lot of market imperfections on the product side.[8]

While the conclusion that factors may not be receiving incomes equivalent to their marginal products can be held to doubt, it is not necessarily true that the divergence we observe is completely due to market imperfections.

It is first essential to disaggregate value added, since this is essentially the analogous "output" of the indus-

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[8] On this, see G.P. Sicat and A.S. Maminta (1968, forthcoming).

Table 4.13. ACTUAL AND ESTIMATED FACTOR-SHARE RATIOS AND CORRECTION FACTOR FOR "IDEAL" DISTRIBUTION OF OUTPUT

ISIC Code	Industry	Recorded r/w	"Best" α_K/α_L	λ
20	Manufactured Food	4.263	1.198	0.281
21	Beverages	3.762	26.027	6.944
22	Tobacco	3.167	1.304	0.443
23	Textiles	1.174	0.362	0.308
24	Footwear & apparel	1.041	0.346	0.332
25	Wood & cork	1.041	0.567	0.545
26	Furniture & fixtures	1.041	1.801	1.730
27	Paper products	2.571	0.348	0.135
28	Printed & published materials	0.852	0.328	0.385
29	Leather products	1.326	0.927	0.699
30	Rubber products	2.846	1.183	0.416
31	Chemical products	3.348	0.420	0.125
33	Non-metallic mineral	2.448	1.083	0.442
34	Basic metal	1.703	0.428	0.251
35	Metal products	1.941	0.808	0.416
36	Machinery, non-electric	1.941	0.439	0.226
37	Electrical machinery	2.030	0.580	0.286
38	Transportation	1.222	0.421	0.344

tries in question. The value added statistics are "gross" rather than "net." Therefore, they include estimates of depreciation. In addition, taxes and interest payments on borrowed funds are part of the gross value added. The remaining is the net rental on the capital goods and any surplus may be truly called profits accruing to owners of the capital goods. The breakdown of all these components are not known from the Surveys of Manufactures, but some adjustments may in fact account for the excessive share of the capital input. So, we have lumped together all value added which is not received by labor as the equivalent of gross capital rental. Another important reason is that, being what they are, surveys are not able to take full account of the return to labor inputs which do not receive any imputed wages. This type of activity would be more rampant among noncorporate enterprises and among corporate enterprises which are largely family-owned. The bias of surveys is to cause the recording of any value added not directly paid to labor as a return to capital. Since the same amount of value added is under consideration, the recorded ratio of the return to capital to wages would be overestimated.

The above explanation is deficient. The estimated production function may be interpreted as an average production function attempts to explain the variations of output.

As shown in the last column of Table 4.11, the estimates of $\bar{R}^2$ shows that the linear regressions of output per man on capital per man, which yielded the estimates of α_K, are not able to explain a great deal of the variations of output per man, although as we have emphasized, the slopes, α_K, were statistically different from zero. The presence of large unexplained residuals from the estimated regression is an indication of the wide difference between certain groups of industries which are "off" the estimated capital shares. Indeed, it may be advanced that some observations have relatively more weight than others when viewed in terms of the total industry. But in a regression, one observation is equally weighty as the other observation and therefore some bias may result in the estimates of α_K. The direction of this bias is hard to predict.

In any case, the divergence from "ideal" factor-shares which is measured by λ is obviously the combined result of the different reasons we have enumerated. Even if it is possible to take out the biases in measurements due to census accounting and regression bias, the divergence due to market imperfections must still be large. While this fact limits some of the usefulness of production function estimates, they should not deter us from seeking approximate answers to the vital question of the structure of production. In terms of this search for the answer to the structure of production in Philippine manufacturing, the varied results presented in this chapter tend to support the appropriateness of a Cobb-Douglas production function specification, in spite of our qualifications as to its failure to predict the distributional implications. This becomes apparent especially after a reading of the material reported in the subsequent chapter.

Chapter 5. CES PRODUCTION FUNCTIONS

Introduction

The family of production functions with constant elasticity of substitution between the two inputs, which was introduced and analyzed fully in the classic paper of Arrow, Chenery, Minhas, and Solow, or ACMS (1961), has aroused many studies since its first appearance. This chapter will report the estimates of this production function by two-digit Philippine manufacturing.

In Chapter 2, it is pointed out that the CES production function,

$$Q = \gamma \{ \delta K^{-\rho} + (1-\delta)L^{-\rho}\}^{1/\rho}$$

where γ is a technological parameter, δ as distribution parameter, and ρ the factor substitution parameter, is estimated by the equation

$$\ln Q/L = \ln A + b \ln W + e.$$

The special assumptions of this production function are that there is competitive factor pricing so that factor incomes are equated with their marginal products and that the function is homogeneous of degree one.

Under special assumptions that the efficiency of the industry is not affected by the wage rate,[1] the estimate of

[1] ACMS (1961), see esp. pp. 136-238.

b corresponds to the elasticity of substitution, σ. This elasticity is related to the substitution parameter, ρ since, as already pointed out, $b = \sigma = 1/(1+\rho)$, or $\rho = (1/\sigma) - 1$.

As mentioned in Chapter 2, this production function admits of wider variability in the possibilities of substitution between capital and labor than either the Cobb-Douglas or the Harrod-Domar-Leontief fixed proportions production functions. The Cobb-Douglas production function, which was analyzed in the previous chapter, has unit elasticity of substitution between the two factors, while the fixed proportions production function admits of no possibility of substitution, in short, $\sigma = 0$.

At least, the value of σ should be estimated rather than assumed. Most findings about the elasticity of substitution has placed its value between 0 and 1, but closer to unity. It is first essential, before comparing Philippine results to these findings, to report the estimates for the Philippines.

Estimates of CES Production Functions

As mentioned in Chapter 3, there are two definitions used for wages. The first concept of wages (W_1) includes only those for production workers, the second (W_2) includes all wages paid by the establishments. In addition, output is measured either as gross sales on value added. Following the arguments stressed in Chapter 4, it matters little if gross sales or value added is used, because if the two are related, as in general they are, they can affect only the constant terms of the regression estimates.[2] For labor, all employed workers were used. Again, on this score, we have reasoned in Chapter 3 that the ratio of production to total employed workers do not vary widely within one industry group and therefore could not affect the overall results significantly.

We lay out the different equations used in the derivation of the different estimates of the elasticity of substitution below:

(a) In terms of observations used, aggregates per employment size of establishments and random samples of establishments, in the manner done in the previous chapters were used.

(b) In terms of the concepts used, gross sales per man and value added per man were alternatively utilized in the dependent variable. In the independent variable, yearly payrolls per man were used.

These are the equations:

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[2]See above, pp. 4-8 to 4-18.

(5.1) $$\ln V/L = \ln A + b_{w1} \ln W_1 + e$$

(5.2) $$\ln G/L = \ln B + b^*_{w1} \ln W_1 + e$$

(5.3) $$\ln V/L = \ln A + b_{w2} \ln W_2 + e$$

(5.4) $$\ln G/L = \ln B + b^*_{w2} \ln W_2 + e,$$

where V is value added, G gross sales, L total employment, W_1 yearly wages of production related workers, W_2 yearly wages of all workers, and e a random term with mean zero and constant variance.

Since we are interested largely in the magnitude of the elasticity of substitution $\underline{b}$, we shall report the results for this magnitude only. Table 5.1 and 5.2 report all the estimates of $\underline{b}$ which were statistically significant.[3] The last columns of both tables is a simple arithmetic mean of the coefficients; the numbers in parentheses are the total number of coefficients which are averaged. The astonishing results from all the regressions performed is the relatively high value of the estimate for $\underline{b}$. Table 5.3 all the estimates in terms of whether they exceed the value 1. (We note, in reminder that, $\underline{b} = \sigma = 1$ is the Cobb-Douglas case.)

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[3] Many are significant at the 1 and 5 per cent probability level; a few are significant at the 10 per cent level.

Table 5.1. CES PRODUCTION FUNCTIONS ESTIMATES FOR THE ELASTICITY OF SUBSTITUTION BASED ON AGGREGATED ESTABLISHMENTS

ISIC Code	Industry	Value Added		Gross Sales		Averag
		b_{w1}	b_{w2}	b^*_{w1}	b^*_{w2}	
20	Manufactured Food	1.696 (0.281)	1.698 (0.317)	0.711 (0.289)	0.939 (0.256)	1.26 (4)
21	Beverages	1.626 (0.129)	1.357 (0.214)	1.253 (0.123)	0.979 (0.203)	1.30 (4)
22	Tobacco	1.585 (0.432)	1.499 (0.320)	1.604 (0.296)	1.427 (0.223)	1.52 (4)
23	Textiles	n.s.	n.s.	n.s.	n.s.	
24	Footwear & apparel	0.542 (0.268)	0.512 (0.196)	n.s.	0.673 (0.468)	0.57 (3)
25	Wood and cork	n.s.	0.631 (0.468)	n.s.	n.s.	0.63 (1)
26	Furniture & fixtures	1.390 (0.230)	1.256 (0.133)	1.806 (0.262)	1.617 (0.142)	1.51 (4)
27	Paper products	n.s.	1.967 (0.689)	n.s.	1.594 (0.603)	1.78 (2)
28	Printed & published mats.	0.540 (0.238)	n.s.	0.368 (0.222)	n.s.	0.45 (2)
29	Leather products	1.101 (0.386)	n.s.	1.259 (0.246)	0.836 (0.760)	1.06 (3)
30	Rubber products	1.533 (0.379)	1.726 (0.208)	1.325 (0.372)	1.560 (0.198)	1.53 (4)
31	Chemical products	1.477 (0.461)	1.324 (0.336)	0.874 (0.575)	0.624 (0.473)	1.07 (4)
33	Non-metallic mineral	n.s.	2.035 (0.309)	n.s.	1.711 (0.304)	1.87 (2)
34	Basic metal	0.485 (0.336)	1.362 (0.409)	n.s.	0.955 (0.399)	0.93 (3)
35	Metal products	1.578 (0.614)	0.875 (0.626)	1.395 (0.944)	n.s.	1.2 (3
36	Machinery, non-electric	n.s.	1.488 (0.644)	n.s.	0.903 (0.744)	1.1 (2
37	Electric machinery	n.s.	1.216 (0.432)	n.s.	0.959 (0.550)	1.0 (2
38	Transportation	0.453 (0.195)	0.674 (0.313)	0.794 (0.307)	1.158 (0.500)	0.7 (4

Standard errors of coefficient in parentheses.

Number in parentheses under average is number of estimates from which average is derived.

Table 5.2. CES PRODUCTION FUNCTIONS: ESTIMATES FOR THE ELASTICITY OF SUBSTITUTION BASED ON SAMPLED ESTABLISHMENTS

ISIC Code	Industry	Value Added		Gross Sales		Avera
		b_{w1}	b_{w2}	b^*_{w1}	b^*_{w2}	
20	Manufactured Food	1.413	1.709	1.369	1.396	1.47
		(0.442)	(0.264)	(0.300)	(0.211)	(4)
21	Beverages	0.953	1.146	0.763	0.809	0.91
		(0.165)	(0.226)	(0.117)	(0.203)	(4)
22	Tobacco	1.528	1.564	1.697	1.662	1.61
		(0.406)	(0.318)	(0.377)	(0.306)	(4)
23	Textiles	n.s.	n.s.	n.s.	0.444	0.44
					(0.375)	(1)
24	Footwear & apparel	0.536	0.591	0.641	0.635	0.60
		(0.234)	(0.192)	(0.291)	(0.260)	(4)
25	Wood and cork	0.899	1.166	0.688	0.903	0.91
		(0.220)	(0.117)	(0.193)	(0.128)	(4)
26	Furniture & fixtures	1.235	1.307	1.372	1.453	1.34
		(0.269)	(0.177)	(0.291)	(0.183)	(4)
27	Paper products	0.565	1.184	0.875	1.297	0.98
		(0.482)	(0.336)	(0.402)	(0.247)	(4)
28	Printed & published mats.	0.818	0.910	0.741	1.341	0.95
		(0.217)	(0.600)	(0.269)	(0.595)	(4
29	Leather products	1.218	n.s.	1.105	0.556	0.90
		(0.410)		(0.312)	(0.600)	(3
30	Rubber products	1.559	1.798	1.519	1.609	1.6
		(0.592)	(0.272)	(0.525)	(0.284)	(4
31	Chemical products	n.s.	1.500	n.s.	0.726	1.1
			(0.400)		(0.588)	(2
33	Non-metallic mineral	1.191	1.249	0.939	0.964	1.0
		(0.256)	(0.235)	(0.269)	(0.264)	(4
34	Basic metal	n.s.	0.974	n.s.	n.s.	0.9
			(0.723)			(1
35	Metal products	0.810	1.827	n.s.	1.664	1.4
		(0.753)	(0.612)		(0.617)	(3
36	Machinery, non-electric	1.143	1.045	1.060	0.746	0.9
		(0.706)	(0.522)	(0.762)	(0.604)	(4
37	Electric machinery	0.492	0.796	n.s.	n.s.	0.6
		(0.384)	(0.590)			(2
38	Transportation	n.s.	0.608	n.s.	0.838	0.7
			(0.321)		(0.481)	(2

Standard errors of coefficients in parentheses.

Number in parentheses under average is number of estimates from which average is derived.

Table 5.3. ESTIMATES OF ELASTICITY OF SUBSTITUTION RELATIVE TO UNITARY CES-VALUE

ISIC Code	Industry	Number of estimates of $\sigma = b \geq 1$	From a total number of significant estimates	Average Estimates
20	Manufactured Food	6	8	1.366
21	Beverages	4	8	1.111
22	Tobacco	8	8	1.571
23	Textiles	0	1	0.444
24	Footwear and apparel	0	7	0.590
25	Wood and cork	1	5	0.857
26	Furniture & fixtures	8	8	1.430
27	Paper products	3	6	1.247
28	Printed & published materials	1	6	0.786
29	Leather products	4	6	1.012
30	Rubber products	8	8	1.578
31	Chemical products	2	6	1.088
33	Non-metallic mineral	4	6	1.348
34	Basic metal	1	4	0.944
35	Metal products	4	6	1.358
36	Machinery, non-electric	4	6	1.064
37	Electrical machinery	1	5	0.866
38	Transportation	1	5	0.754
	Total Estimates	60	109	

*Number of estimates from which average is derived taken from previous column.

Out of a total of 109 CES regressions, 60 estimates had b > 1. This represents 55 per cent of all significant estimates arrived at. When we examine the average value of the estimates, 9 2-digit industries had estimates greater than 1. Those industries with less than 1 elasticity of substitution are: textiles (ISIC 23), foctwear & apparel (ISIC 24), wood and cork (ISIC 25), printing (ISIC 28), basic metal (ISIC 34), electric machinery (ISIC 37), and transportation (ISIC 38). Industries which had only few possible estimates which are significant from a statistical viewpoint are very few. In fact, only textiles (ISIC 23) had one relatively good estimate. The other industry, which had only 1/2 of possible estimates coming out relatively significant, is basic metals (ISIC 34).

A comparison of estimates of the elasticity of substitution drawn from aggregates for employment sizes and from a sampling of these employment sizes are easily compared by drawing a scatter of the averages of the estimates. It is seen from this information that the two estimates tend to cluster at the same value, with significant exception of paper (ISIC 27), leather (ISIC 28) and nonmetallic mineral (ISIC 33). The scatter also dramatizes the relatively high values of the elasticity of substitution found for most of two-digit manufacturing.

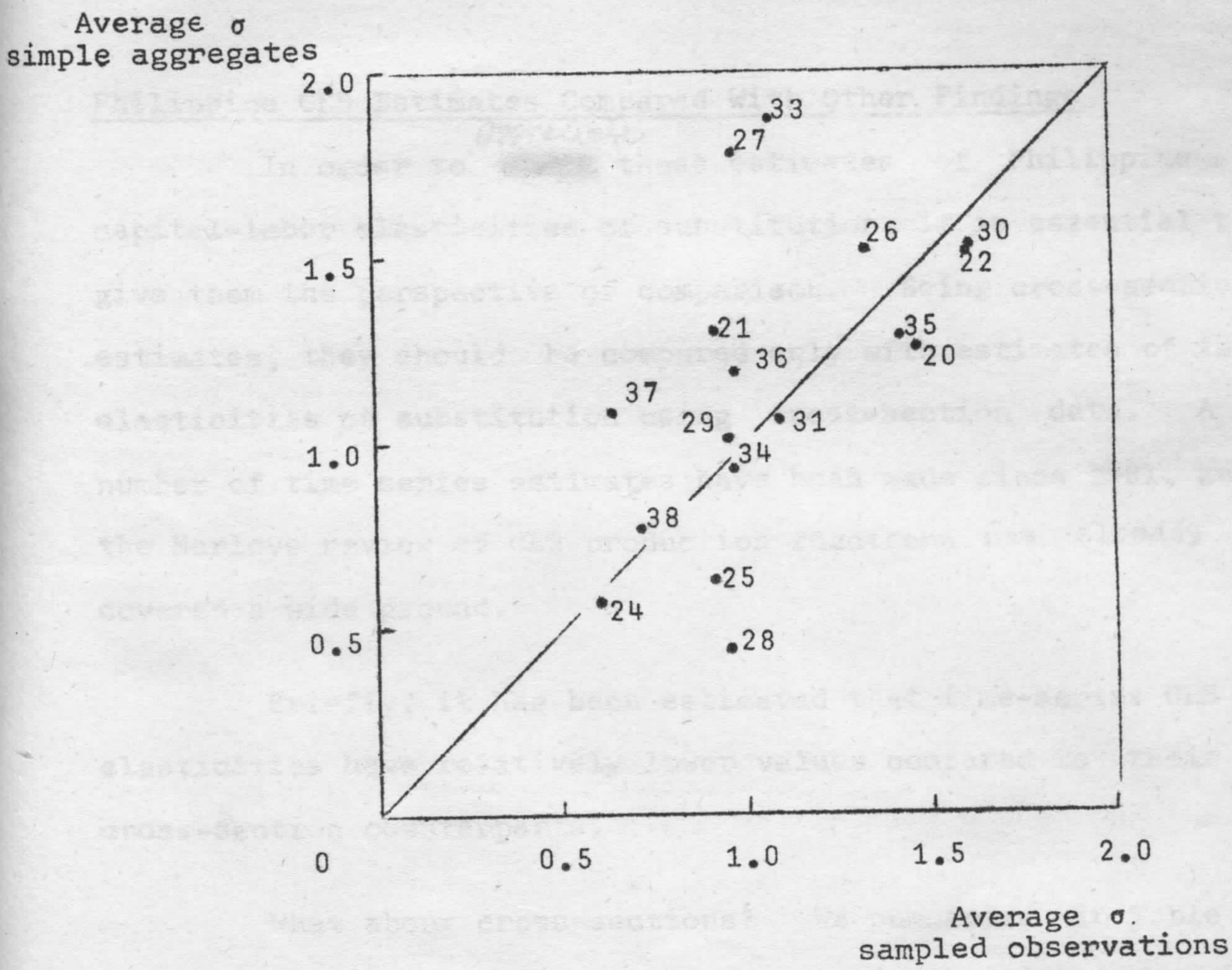

Figure 5.1. Comparison of Average Elasticities of Substitution By Industry

Thus, in general, most of the estimates for the elasticity of substitution in Philippine manufacturing are greater than one. In the next section, we shall review the estimates found in other studies.

Philippine CES Estimates Compared With Other Findings

In order to [redacted] appreciate these estimates of Philippine capital-labor elasticities of substitution, it is essential to give them the perspective of comparison. Being cross-section estimates, they should be compared only with estimates of the elasticities of substitution using cross-section data. A number of time series estimates have been made since 1961, and the Nerlove review of CES production functions has already covered a wide ground.

Briefly, it has been estimated that time-series CES elasticities have relatively lower values compared to their cross-section counterparts.

What about cross-sections? We summarize in Table 5.4. the estimates of CES functions from international data by reproducing, with minor changes, the table prepared by Nerlove comparing these estimates. The original estimates of ACMS were found to be in general less than one, although they are relatively close to unity. A reestimation by Fuchs (1963) of

Table 5.4. INTERCOUNTRY ESTIMATES OF THE ELASTICITY OF SUBSTITUTION FOR TWO-DIGIT MANUFACTURING STUDY

ISIC Code	Industry	Ave. of Murata-Arrow (Data for 1953-56 and 1957-59)	ACMS (1961)	Fuc (196
20	Food	(	0.83[1]	1.0
		(0.724[a]		
22	Tobacco	(	0.75	1.2
23	Textiles	0.810	0.80[2]	0.9
24	Apparel & related materials	0.732	-	-
25	Wood & lumber	(	0.86	1.0
		(0.868		
26	Furniture	(	0.89	1.0
27	Paper	0.846	0.97	0.9
28	Printing & publishing	0.881	0.87	1.0
29	Leather	0.705	0.86	0.9
30	Rubber	0.798	-	-
31	Chemicals	0.836	0.86[3]	1.0
32	Petroleum and coal	-	-	-
33	Stone, clay, glass	0.853	0.94[4]	1.0
34	Primary metals	0.864	0.91[5]	0.8
35	Metal products	0.920	0.90	1.0
37	Electrical machinery	-	0.87	1.0

[a]Includes beverages & tobacco.

[1]Average of Dairy products, Fruits & vegetables, Grain & mill product Bakery products, and sugar.

[2]Average of Spinning & weaving and Knitting mills.

[3]Average of Basic chemicals, Miscellaneous chemicals, Fats & Oils.

[4]Average of clay, Glass, Ceramics & Cement

[5]Average of Iron & steel and nonferrous metals

the same data, with the exception that some account was taken of the degree of development of the countries whose observations were used with the application of dummy variables, led to estimates which are much closer to unity. An attempt by Murata and Arrow (1965) have reconfirmed the results obtained originally by ACMS.

However, the results of Solow (1964), utilizing cross-section data by US regions in 1956, and Griliches (1967), who used regional data in 1958, should clear the gound for comparison. Table 5.5 reproduces the major results for similar equations.[4] Except for some industries in which Solow derived some statistically nonsignificant estimates (these were: chemicals, stone, clay (nonmetallic mineral), electrical machinery, and transport equipment), the Solow results are close to those obtained by Griliches. On the other hand, the average elasticities of substitution by industries directly obtained for the Philippines do not appear to be any smaller compared to those obtained for American manufacturing using relatively the same aggregation. This may appear surprising especially because it is generally believed that the less developed economies have much narrower degrees of capital-labor substitution.[5]

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[4]Griliches (1967a) attempted to use other estimating equations, because he combined two cross-sections and thereby enabled him to use lagged functions for the CES function.

[5]For instance, see R.S. Eckaus (1955).

Table 5.5. CROSS-SECTION ESTIMATES OF THE ELASTICITY OF SUBSTITUTION BETWEEN CAPITAL AND LABOR IN MANUFACTURING INDUSTRIES

ISIC Code	Industry	US Solow (1963)	US Griliches (1967a)	Philippin[es] Average (Sicat)
20	Food	0.69	0.98	1.37
21	Beverages	-		1.11
22	Tobacco	1.96		1.57
23	Textile mill products	1.27	0.94	0.44
24	Apparel & related products	1.01	1.06	0.60
25	Lumber & wood products	0.99	1.07	0.86
26	Furniture & fixtures	1.12	1.04	1.43
27	Pulp, paper & products	1.77	1.67	1.25
28	Printing & publishing	1.02	0.83	0.79
29	Leather products	0.89	0.84	1.01
30	Rubber products	1.48	1.28	1.58
31	Chemicals & products	0.14	0.71	1.09
32	Petroleum & coal	1.45	-	-
33	Stone, clay, glass	0.32	0.91	1.35
34	Primary metal products	1.87	1.41	0.94
35	Fabricated metal products	0.80	0.85	1.36
36	Non-electrical machinery	0.64	1.24	1.06
37	Electrical machinery	0.37	0.66	0.87
38	Transportation equipment	0.06	0.91	0.75
39	Instruments & related products	1.59	0.75	-

Source: R.M. Solow (1964), Griliches (1967a).

Table 5.3, above.

We shall now attempt to provide some explanation for the differences and similarities of the estimates just discussed.

Upward Bias in CES Estimates

As early as the original paper on CES production functions of ACMS, it was recognized that estimates of the elasticity of substitution from standard regressions have upward bias. ACMS (1960, esp. pp. 236-7) have suggested that if efficiency levels of the observations vary directly with the wage rate -- as in general they should -- the elasticity of substitution would no longer be equal to b. In this case,

$$\sigma = \frac{b - e}{1 - e}$$

where $e(\geq 0)$ is an elasticity parameter relating the wage rate to the efficiency parameter, γ. It is clear that b is upward biased here if $1 > b > e$. However, if $b>1$, as the case is for the estimates in this study, the elasticity of substitution will still be higher!

Thus, the important question is whether there is an upward bias for estimates of b even when they exceed unity. Many explanations have been made, and the most comprehensive discussion of this can be found in the review of the literature on CES production functions by Nerlove and the discussion that followed, which are all reported in the conference volume on production functions, edited by Brown (1967).

Most of the estimates of elasticities of substitution are derived from regional observations of value added per man and wage rates. The regions may be countries, as in the case of the ACMS (1961) and the Murata-Arrow (1965) or as in the case of cross-section studies of the CES production functions in the United States, observations per state.[6] Thus, the discussion of the upward bias has been centered around the regional observations used.

We recall that the observations in this study are establishments classified by asset sizes. It is unfortunate that we are not able to parallel estimates of elasticities of substitution with any studies utilizing the same concepts. It was impossible to utilize for the Philippines regional estimates of the needed observations, because about 50 per cent of the relatively small manufacturing sector (about 1/5 of total national income) is located in one region. Moreover, no detailed tabulations of regional manufacturing statistics are available. Nonetheless, these discussion have some bearing on the nature of the estimates we have derived.

As we have already reported in the previous section, (1) the studies of international cross-sections have yielded generally less than 1 elasticities of substitution and (2) the cross-section studies within single countries have yielded higher values of the elasticities of substitution compared to their international counterparts. The results of studies of Solow (1964), Minasian (1961), Hildebrand & Liu (1964), among others, have confirmed this. In a number of cases, these estimates have exceeded unity.

It will be useful to review some of the more important explanations for the upward bias in CES estimates. Eisner (1963) has argued this using reasoning similar to the permanent income hypothesis relating to consumption function estimates. Following McKinnon (1963) who suggested that product prices influence the estimate of the elasticity of substitution, Nerlove (1967) has shown that while the equation

(5.5) $$\log V/L = a + b \log W$$

is in real terms, the elasticity of substitution is often estimated as

(5.6) $$\log pV/L = a' + b' \log pW,$$

as Solow (1964) himself did, where V and W are in <u>real</u> measures and p is a price index. The equivalent expression for (5.6), however, is