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CONSTRAINED MAXIMIZATION AND DECISION RULES IN GOVERNMENT: AN ANALYSIS OF AN INVESTMENT ALLOCATION FORMULA FOR THE PHILIPPINES

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In the nineteen sixties, applied microeconomic analysis is enjoying a boom, and its practitioners are luxuriating in a sellers' market. For years it had seemed that the main functions served by the teaching of micro-economic theory in universities were to provide students with a "mental discipline" and a body of information which they as teachers could pass on to the next generation of students. Recently, however, cost-benefit analysis and its close relatives, systems analysis, and cost-effectiveness analysis have ~~belied that narrow view~~ forced us all to realize that microeconomics can be useful. Deriving and applying the implications of constrained maximization--and that primarily is what microeconomic analysis has consisted of[1/] -- has become fashionable and valuable in government as well as private decision making.

Surprisingly, seldom until very recently have attempts been made to specify actual objective functions

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1/ Which is not to deny the existence or importance of a relatively small body of literature on satisficing and other non-maximizing theories of choice.

of decision makers. Economic analysis of the firm has generally assumed profit or sales maximization to be the objective function of an entrepreneur -- that is, very simple maximands. On the other hand, developers of the theory of the household have recognized the extreme complexity of an individual's pattern of wants, and by and large have been content to posit certain characteristics of an individual's utility function (e.g., transitivity of preferences, diminishing marginal rate of substitution, etc.) rather than tackle the very likely impossible task of specifying the function. But a common characteristic of the development of the theory of the firm and that of the household is that in both cases the respective objective functions are assumed to be maximized.[2/] Competition and the survival of the fittest, it is argued, lead to profit maximization, at least by the survivors. Modern developments in consumer theory -- particularly the theory of revealed preference -- depend on the utility maximizing assumption.

[2/] I do not mean to suggest that profit and sales maximization are the only objective functions which have been considered by economists, but rather that it has generally been assumed that the firm or household does in fact maximize its objective function, whatever it may be.

The purpose of this paper is not to question the appropriateness of the maximizing assumption for the firm or the household. The intent here is, rather, to show that economists, in our eagerness to apply our analytical tools to government problems, have tended to overlook the possibility that objective functions in government may not "automatically" be maximized. I submit that in a society which leaves some range of choice to the discretion of its decision makers, there is no guarantee that even with the best intentions those decision makers will maximize their political objective functions.[3/] This should not be surprising. If the utility function of an individual is too complicated to be specified and maximized by an "outside" analyst, even with the impressive mathematical and computational means now available, why should we expect a politician or bureaucrat to accomplish this feat unaided by technical

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[3/] William A. Niskanen has argued that bureaucratic decision makers typically misallocate resources, since they act in their own interests (perhaps maximizing their ~~own~~ agency budgets) which may not coincide with the interests of the society in general. The thesis of the present paper does not necessarily conflict with Niskanen's. Even if bureaucrats do act in their own interest rather than society's, there may be some issues on which they are indifferent as to outcomes, and others where they have an interest but the outcomes of their actions are difficult to ascertain. In

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analytical tools? Often he may have neither the market feedback experienced by a firm, nor the introspectively perceived uneasiness of an individual dissatisfied with the way he spent last week's paycheck.

Explicit budget allocation and project ranking formulas are widely used in government. In the economist's analytical framework these can be interpreted as decision rules derived from a constrained maximization process. But these formulas often contain ten, twenty, or more variables--meaning that there are at least that many variables in the maximand and constraints. Then to assume that the formula does in fact represent the maximizing of the decision maker's objective function, is to assume that either superior logic, or "uneasiness"[i] or "competitive" political pressures led to that result. The former possibility is unlikely since it would attribute to the unaided human mind an ability (to maximize subject to constraints an involved analytical expression) often not possessed even by humans with the aid of

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either case ~~(as argued here)~~ the implications of their actions may coincide with neither their intentions nor those of society. Niskanen's argument and that presented here suggest two different explanations for the possibility that government decision makers misallocate resources. Cf., Niskanen, "The Peculiar Economics of Bureaucracy," American Economic Association Papers and Proceedings, May 1968.

sophisticated mathematical and computational techniques.) Juggling slopes on a hyper-surface in twenty dimensions is hard work. There remain the possibilities that the forces assumed to be present in the firm and the consumer -- namely the "competition" and "uneasiness" referred to above -- produce an optimum in the case of the government. Elsewhere I have shown evidence which indicates that these forces are not always operative in the United States government.[4/] Following is a brief analysis of a project ranking index devised by a Philippine government body. It is shown that some implications of the formula are possibly inconsistent with the intentions of the designers -- an indication that their intentions are not being carried out, their objectives not being maximized. If this situation is widespread, it suggests the correspondingly widespread existence of unrecognized non-achievement of Pareto optimality.

An Investment Allocation Formula

The Philippine Investment Incentives Act of 1967 set up a Board of Investments to "encourage Filipino and foreign investments," as well as allocate investment funds

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[4/] "On Budget Allocation in Government Agencies," Review of Social Economy, March 1967.

and provide incentives to deserving areas. This allocation was to take into consideration eight goals of the society, ranging from accelerating "the sound development of the national economy in consonance with the principles of economic nationalism," to providing for an "equitable distribution of wealth."[5/] In an ambitious and impressive attack on this problem the Board of Investments in early 1968 published an Investment Priorities Plan, and an index or formula used to "refine" the selection to suit the framework of the Plan."[6/]

Projects may be ranked in order of value received in the formula, a complex index involving 15 variables. An optimum allocation would presumably be reached by allocating funds to projects in order of their index value

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5/ "An Act Prescribing Incentives and Guarantees to Investments in the Philippines, Creating a Board of Investments, Appropriating the Necessary Funds Therefor and for Other Purposes," R.A. 5186, Sixth Congress of the Philippines, Fifth Special Session, 1967, pp. 1-2.

6/ "Rationales for the Investment Priorities Plan: Vol. 1, Factors Considered in the Plan," Board of Investment, Office of the President, Republic of the Philippines, March 15, 1968, p. ii.

with the highest valued project funded first. (Lumpiness of project cost, the so-called knapsack-problem, can be handled by standard mathematical programming techniques.)

A question to be asked is whether allocation by means of the index would be a true reflection of the intentions of its designers. If the conjecture outlined above is correct, one might expect that rates of substitution implied by the index could conflict with those intentions. Derivation of these rates of substitution is a straightforward matter. Holding the index constant at a given value, the rate of substitution between any two variables of the formula is simply the partial derivative of one of these with respect to the other. If, then, creation of foreign exchange earnings and of jobs are both included in the index, the rate of substitution between these two indicates the amount of incremental foreign exchange earnings which is implied to be equivalent to the creation of an additional job. The derivation of this and other tradeoffs follows (after explicit statement of the index).

The "index of productive output" is:

$$I_{p\,0} = 30X_1 + 30X_2 + 15X_3 + 15X_4 + 10X_5,$$ [7]

where[8] X_1 is foreign exchange earnings and savings associated with a project, per peso invested in the project;

X_2 is average yearly (undiscounted) "social return" in pesos, per peso invested;

X_3 is jobs created per peso of investment;

X_4 is a measure of forward-backward linkage effects; and

X_5 is an index of the degree of "insufficiency" of the good produced by the project.

Each of the five X_i is then a sub-index involving several variables, so the total index becomes (after "normalizing" as described in footnote 8):

$$I_{p\,0} = \frac{30\left[\left|\overline{E}-(M+A)\right| + \left|\overline{P}-(M+A)\right|\right]}{3.061K} + \frac{30}{76KN}\sum_{i=1}^{N}(R_i - C_i)$$

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[7] Cf., "Investment Priorities Plan," Board of Investments, Office of the President, Republic of the Philippines, March 15, 1968, p. 21.

[8] Each of the variables appears in the index not in absolute terms but as a percentage of the largest value of that variable among all the projects considered. Thus, e.g., X_1, the measure of foreign exchange earned (F) per peso of investment (K), enters the index not as $\frac{F}{K}$, but as F/K (3.061), where 3.061 is the highest value of F/K attained by any of the 59 projects considered.

$$+ \frac{15}{547} \frac{W}{K}$$

$$+ \frac{15}{156} \left[\frac{B}{T} + \frac{S}{D} \right]$$

$$+ \frac{10\ H(D-J)}{1.0\ D} ,$$

or

$$Ip\ 0 = 9.8\frac{F}{K} + .3947\frac{G}{K} + 27,422\frac{W}{K} + .096L + 10I.$$

All of the variables are described in the appendix to this paper, but those discussed below are

F: net foreign exchange earnings and savings

K: total capital investment

G: average annual net social return

W: jobs created by the project

The rate of substitution between F and W

Consider, then, the partial derivative of F with respect to W.

$$\frac{\partial F}{\partial W} = \frac{-\partial 0}{\partial W} \Big/ \frac{\partial 0}{\partial F} = -₱2798$$

The rate of substitution between foreign exchange earnings and jobs created is ₱2798. Two projects would receive the same score if – though alike in every other respect – one

of them created 1 more job and the other produced ₱2798 more foreign exchange. The index implies willingness to trade a possible additional job in the society for 2798 additional pesos foreign exchange – or more precisely, indifference between the two. It is the responsibility of the Board of Investments – among others – to decide whether that value judgment is appropriate. It is merely presented here as an implication of the index.

The rate of substitution between K and W

Given the willingness to exchange jobs for foreign exchange as indicated in the previous paragraph, it is of interest to determine the tradeoff between capital and jobs. Here,

$$\frac{\partial K}{\partial W} = \frac{27,422K}{9.8F + .3947G + 27,422W}$$

This rate is not constant as was that between F and W. For the 55 "desirable" projects ranked by the BOI, $\partial K/\partial W$ ranges from 609 for the project ranked first, to 672 for the 30th project, to 3545 for the last, the marginal project. On the margin, one job is as ~~useful~~ valuable, as ~~valuable~~ desirable to the society, as ₱3545 of capital.

The rate of substitution between G and W

If the society is willing to exchange a possible job for ₱3545 of capital, what rate of substitution is there between G (another peso figure, indicating average annual social return from a project) and W?

$$\frac{\partial G}{\partial W} = -69,475$$

Whereas a job is not valued highly vis-a-vis capital, it is considered the equivalent of ₱69,475 in returns from the project, per year for the life of the project.

The rate of substitution between G and K

This last rather surprising implication suggests another of some interest — the tradeoff between G and K.

$$\frac{\partial G}{\partial K} = \frac{9.8F + .3947G + 27,422W}{.3947}$$

This quantity varies from 114 for the highest ranked project, to ~~73~~ 20 for the lowest, or an implicit willingness on the margin to exchange ~~₱73~~ ₱20 of social return per year for one peso of capital now. ~~That~~ This ~~rate is not 1,~~ may indicates either unawareness that the index implies the higher figures, or possibly a political judgment that it is very difficult to channel resources in the Philippines into capital

investment.[9]

The rate of substitution between F and K

Clearly, both foreign exchange and capital are valued highly, but what is their relative valuation?

$$\frac{\partial F}{\partial K} = \frac{9.8F + .3947G + 27,422W}{9.8K}$$

This time the rate varies from 4.6 for the first project, to .8 for the last. When the first pesos of investment are expended, a peso is the equivalent of ₱4.60 of foreign exchange. But when the last project is funded the rate becomes 80 centavos of foreign exchange per peso of capital.

The rate of substitution between G and F

Finally (though several other rates of substitution could be evaluated), there is the question of the rate at which the society is willing to trade social returns for foreign exchange. Here,

$$\frac{\partial G}{\partial F} = -24.83$$

A willingness is indicated to exchange ₱24.83 of social product per year for the life of the project for one peso

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[9] This rate and the others relating two variables both measured in peso terms do not have values of 1 raises the so-called commensurability problem which is discussed below.

of foreign exchange sometime during the life of the project. The relatively high valuation on foreign exchange implied by the index is again born out.

Marginal Rates of Substitution for the Index

$$0 = 9.8\frac{F}{K} + .3947\frac{G}{K} + 27,422\frac{W}{K} + .096L + 10I \text{ [10/]}$$

Rate of substitution \ Project rank	1	30	55
$\frac{\partial F}{\partial W}$	- 2,798.16	- 2,798.16	- 2,798.16
$\frac{\partial K}{\partial W}$	609.91	671.63	3,545.28
$\frac{\partial G}{\partial W}$	-69,475.53	-69,475.53	-69,475.53
$\frac{\partial G}{\partial K}$	113.81	73.33	19.59
$\frac{\partial F}{\partial K}$	4.59	2.95	0.79
$\frac{\partial G}{\partial F}$	- 24.83	- 24.83	- 24.83

10/ See page 9 for definitions of the variables.

Numerous other rates could be evaluated. (In general, if an index or a utility function contains N variables, there are C (N, 2), or $\frac{N!}{2!\,(N-2)!}$ different combinations of two of them at a time.) But the six calculated above should indicate the usefulness of inspecting such tradeoffs when constructing an index.

Two questions remain to be discussed, namely commensurability (why do not all rates comparing magnitudes measured in the same terms have a value of 1?), and integrability (if one has stipulated the desired levels for the rates of substitution, under what circumstances can an index be inferred from these differential equations?).

Commensurability

One of the most important functions of money is to provide a common measure for large numbers of disparate quantities. And yet, in the index under discussion, a peso is not a peso — the index indicates a willingness to exchange ₱24.83 of social return every year (during the life of the project) for one peso of foreign exchange earnings. Although this may be an implication of the index of which its designers were unaware, it could also reflect the judgment that for whatever reason — political pressure

to have a favorable balance of payments, undesirable price and income disturbances associated with adverse balance of payments, etc. - a peso of income enjoyed domestically by a Filipino is not valued as highly by his society as a peso of foreign exchange. A peso, then, may not in fact be a peso. The index makes these two types of pesos commensurable by stating that one is worth 24.83 of the other. Similarly, as between social return and capital, or foreign exchange and capital. The index also, of course, provides relative weights for such items as social return and jobs, which are measured in altogether different units - providing commensurability by the implied judgment that one job is the equivalent of ₱69,475 of social returns. The purpose of an index is to weight incommensurate items, to provide a relationship which makes them commensurable.

The final topic to be discussed, and one of great practical importance, is whether-even if one can state his marginal rates of substitution (i.e., direction of preferences for incremental changes)-an entire index can be inferred or constructed from those rates of substitution. There is no simple answer to this, the integrability problem. But some guidelines can be indicated.

If government analysts and decision makers - or anyone, for that matter - may not be able to juggle fifteen variables simultaneously, they may nevertheless be able to state their marginal preferences. Thus, if there are, say, three relevant variables x, y, z,[11] this would mean enunciating the marginal rates of substitution of each of these for each of the others. In general, each of the rates of substitution may be a function of each of the variables, or

$$\frac{\partial y}{\partial x} = g\ (x, y, z) \quad \frac{\partial y}{\partial z} = h\ (x, y, z) \quad \frac{\partial z}{\partial x} = j\ (x, y, z) =$$

Each of these partial derivatives denotes a tangent gradient to a point on the (supposed) hypersurface, the index. The set of partial derivatives taken together becomes the total differential equation

$$P\,dx + Q\,dy + dz = 0.$$[12]

This is a tangent hyperplane to the surface. It is this differential equation which must be integrable if the index is to be constructed from the individual rates of substitution.

11/Hereafter the BOI symbols are dropped and the more commonly used symbols of the differential calculus adopted.

12/This equation is easily derived from the rates of substitution by noting that $\partial y/\partial x = -P/Q$, etc.

Conditions will be indicated below under which the equation is integrable, but a pessimistic note must be inserted here. Not only are there some circumstances in which this process of building up an index from a set of individual tradeoffs is impossible even in principle, but in many cases, practical computational difficulties can stand in the way of the derivation even where it is potentially calculable.[13]

The desire now is to find an equation

$$f(x, y, z) = k$$

which is the solution, the "integral" of the differential equation. k is the value of the index, so this equation represents an indifference hyper-surface of the index. Allowing all the influences to vary traces out the entire index hyper-surface. For any value of k, the equation indicates equivalent alternative combinations of values of the variables x, y, z. The BOI index represents this kind of relationship. But now the problem is being approached from the opposite direction. The relevant question here is not, "what rates of substitution are implied by an index?", but "is there an index which is implied by a given set of rates of substitution?". Integrability conditions (i.e., conditions

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[13] Indeed, it was almost as much because of the irksomeness of excessive calculations as for expositional purposes that the index analyzed above, and the functions presently under consideration, are shortened and simplified versions of the BOI formula.

under which such a solution does exist) can be determined.[14] An outline of the complete procedure for inferring an index from rates of substitution, and an example including application of the appropriate integrability condition, follow.

Taking three of the BOI variables, let x be foreign exchange earnings, y capital, and z jobs created. Then derivation of an index involves:

(1) Setting the marginal rates of substitution. As an example take the simplest case, where these are constants. Then

$$\partial y/\partial x = k_1 \ , \ \partial z/\partial y = k_2 \ , \ \partial y/\partial x = k_3 = \frac{-k_1}{k_2}$$

(2) Deriving the differential equation of the indifference hypersurface (cf. footnote 12).

Here that becomes

$$P\,dx + Q\,dy + dz = 0$$

or

$$k_1\,dx + k_2\,dy + dz = 0$$

(3) Determining whether that equation is integrable.

The integrability condition is

$$P\left(\partial Q/\partial z\right) + Q\left(\partial P/\partial z\right) + \left(\partial P/\partial y\right) - \left(\partial Q/\partial x\right) = 0$$ [15]

or $k_1 (0) + k_2 (0) + 0 - 0 = 0.$

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[14] Cf., for example, L. M. Kells, Elementary Differential Equations (New York: McGraw Hill, 1932), p. 83 ff., or any differential equations text.

[15] Ibid.

That is, if this condition holds, the equation has a potentially calculable solution. Here it is met; the equation is integrable.

(4) Solving the equation. This can be the most difficult part of the exercise, since there are few general rules of integration. That a solution exists may of little solace if it is difficult to compute. In the trivial case under consideration, the solution of the equation is

$$k_1\,dx + k_2\,dy + dz = 0$$

$$\int k_1\,dx + \int k_2\,dy + \int dz = \text{constant}$$

$$k_1 x + k_2 y + z = k$$

The latter equation is the index, with k determined by the particular values of x, y, and z associated with a project.

Summary and Conclusion

Economists may delude themselves if they assume that government decision makers actually accomplish the maximization of their objective functions. If individuals do not experience the implications of their actions directly, and are not subject to pressures prodding them inexorably

in the "right" direction, there seems to be no guarantee that the implications of their actions are consistent with their intentions.

A particular investment allocation formula devised in the Philippines was chosen to illustrate the difficulty of constructing an index, and the possibilities for improving indices through application of analysis. It is not always possible to construct an index from knowledge of the rates of substitution of its arguments. Where this is possible, the paper outlined a procedure for doing so. This paper constitutes an effort to improve decision making in government by helping bureaucrats narrow the gap between what they want and what they do.

A P P E N D I X

COMPONENTS OF THE INDEX OF PRODUCTIVE OUTPUT[1/]

The index of productive output is

$$O = 30X_1 + 30X_2 + 15X_3 + 15X_4 + 10X_5.$$

Then:

$$X_1 = \frac{1}{K X'_1}\left[E - (M+A) + P - (M+A)\right] = \frac{\frac{F}{K}}{X'_1} = \frac{F}{3.061K}$$

where: F is net foreign exchange earnings;
K is total capital investment;
~~where~~ E is foreign exchange earnings through exports;

P is foreign exchange earnings through import substitution;

M is imported raw materials;

A is 10% per year amortization of imported capital; and

X'_1 is the largest value of F/K for any of the projects (=3.061).

$$X_2 = \frac{\sum_{i=1}^{N}(R_i - C_i)}{KN\ X'_2} = \frac{G}{K\ X'_2} = \frac{G}{76K}$$

where R_i is revenues associated with the project in year i;

C_i is opportunity cost of the project in year i;

K is total capital investment;

N is the life of the project;

G is, then, average annual net social return; and

X'_2 is the largest value of G/K for any of the projects (=76).

1/ This appendix is copied with only slight modifications and additions from the Board of Investment document "Investment Priorities Plan," op. cit., p. 23 ff.

$$X_3 = \frac{W}{K\, X'_3} = \frac{W}{547K}$$

where: W is jobs created by the project;

K is total capital investment; and

X'_3 is the largest value of W/K for any of the projects (=547).

$$X_4 = \left[{}^{B}/_{T} + {}^{S}/_{D} \right] \Big/ X'_4 = \frac{L}{X'_4} = \frac{L}{156}$$

where: B is inter-industry purchases;

T is total production;

S is inter-industry sales;

D is total demand;

L is an index of the extent of forward-backward linkage; and

X'_4 is the largest value of L for any of the projects (=156).

$$X_5 = \frac{H}{X'_5}\left[\frac{D - J}{D}\right] = \frac{I}{X'_5} = I$$

where: H is an "essentially" factor provided by the Board of Investments;

D is total demand;

J is total supply;

I is a composite insufficiency index; and

X'_5 is the largest value of I for any of the projects (=1).

Thus, the index becomes

$$I_{p\,0} = \frac{30F}{3.061K} + \frac{30G}{76K} + \frac{15W}{547K} + \frac{15L}{156} + \frac{10I}{1}$$

$$= \frac{9.8F}{K} + \frac{.3947G}{K} + \frac{27,422W}{K} + .096L + 10I \,.$$