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The present note is an attempt to deduce from simple growth models "optimum" growth rates of the quantity of money under alternative assumptions about technical change. The method used is comparative statics; i.e., only long run equilibrium growth rates are compared. Thus, the effects of the quantity of money and of short run expectation variables on the speed of adjustment are ignored. For this paper's purposes, "optimum" growth rate of the quantity of money is defined as that equilibrium growth rate which, if attained, will preserve price stability over time. Section I considers very briefly the optimum growth rate of the quantity of money in growth models involving *exogenous* technical change. Section II extends the analysis to growth models involving *endogenous* technical change. Section III concludes.

I. Models with Exogenous Technical Change

We need a general expression for the optimum equilibrium growth path of the quantity of money. Assume that, over the long run, the observed real money stock is a unit-homogenous function of output and "efficiency-corrected" population,

$$(1) \quad M/P = G(Q, yN)$$

where M/P is observed real money stock, Q output, yN "efficiency-corrected" population base, and y a technical change multiplier.

Assume that the production function is given by

$$(2) \quad Q = F(K, yN)$$

where K is the capital stock, and F is assumed to be unit-homogenous. It follows that Eq. (1) can be written as

$$(3) \quad [(M/P)/yN] = G[F(k, 1), 1]$$

where $k = K/yN$.

If a long run well-behaved equilibrium exists, there is a determinate equilibrium capital intensity k^* . Plugging this value into Eq. (3), equilibrium real money stock per "efficiency-corrected" head is determined,

$$(4) \quad [(M/P)/yN]^* = G[F(k^*, 1), 1].$$

For Eq. (4) to hold, M/P must grow at the same rate as yN ,

$$(5) \quad \dot{M}/M - \dot{P}/P = \dot{y}/y + \dot{N}/N.$$

If price stability is to be preserved over time, then the quantity of money must grow according to the following,

$$(6) \quad \dot{M}/M = \dot{y}/y + \dot{N}/N.$$

1.1 Basic Solow-Swan Model

I have shown in a paper on a neoclassical version of the Tobin model [19] that the growth rates of all variables

except population are determined by (in fact, are equal to) the sum of the pervasive growth rates of *exogenous* Harrod-neutral technical change and of population [15][16][17][6]. This is no less true of the growth rate of the quantity of money if price stability is to be maintained. This is a special case of Eq. (6), where $\dot{y}/y = \lambda$, and $\dot{N}/N = n$. If the economy is growing at a long run equilibrium rate of 3 per cent, the implication is that the money stock must grow at 3 per cent at stable price levels.

1.2 Variable Bias Models

Variable bias models [3][13][14] essentially are attempts to provide economic explanations why the basic Solow-Swan model and variations on it have long run tendencies to be well-behaved under conditions of constant returns to scale but nonneutral technical change. These models start from the assumption of an *exogenously* given *total* technical change or an *exogenously* positioned technical change frontier. Different technical change multipliers for capital and labor are postulated. The growth rates of these multipliers are then related via a trade-off function. Assuming that the elasticity of substitution < 1 , these "induced innovation" models are able to show that, on maximizing the equilibrium growth rate of output, the economy is induced to allocate zero growth rate to the technical change multiplier for capital and a

determinate, positive growth rate to the technical change multiplier for labor. Once this is granted, these models assume the well-behaved characteristics of the Solow-Swan model. The optimum behavior of the growth rate of the money stock in these models is then identical to that in the Solow-Swan case (1.1).

II. Models with Endogenous Technical Change

The models briefly discussed in the preceding section share a common weakness: they constrain the optimum equilibrium per capita growth rate of the money stock to the exogenous rate of (usually Harrod-neutral) technical change, independent of changes in any of the structural parameters of the models. The models to be discussed in this section attempt to deal with this basic shortcoming.

2.1 Multiples-of-n Models

To illustrate the mechanism behind the multiples-of-n models [18][11][1], consider the following aggregate production function:

$$(7) \quad Q = H(K, N^\mu)$$

where the variables are defined as before and $\mu > 1$. In equilibrium, the optimum growth rate of the money stock is equal to μn . In Uzawa's model, $\mu = 2$. This number is endogenously determined by relating the rate of change in the technical

change multiplier to the present level of technical change and a portion of the population base devoted to increasing technical change. In Arrow's learning-by-doing model, the mechanism for generating technical change is also endogenous. Arrow postulates that the level of technical change is a function of past accumulated experience indicated by the endogenously determined total accumulated resources devoted to increasing technical knowledge, research, and the like. Phelps' extension of Uzawa's model is merely a generalization; in addition to population, Phelps adds capital in the production of technical change.

2.2 Productivity Multiplier Models

Although the multiples-of-n models are, as a class of models, an improvement over the models of Section I in that technical change is now treated as endogenous, other basic defects characterize even these models. First of all, multiples-of-n models imply that the per capita growth rate of output must increase with increases in the population growth rate. Second, they imply that the optimum per capita growth rate of the money stock must be invariant with any change in the structural parameters. The second implication is a radical one; it suggests that optimum monetary policy with respect to the growth rate of the quantity of money should be unaffected by changes in the savings rate, the desired

cash-income ratio, depreciation rate, and the parameters of the production function. Indeed, this proposition is rather surprising. To deal with this problem, the productivity multiplier models were developed. Essentially, the purpose of these models is to get other structural parameters into the act. Thus, the optimum growth of the quantity of money in these models is *not* simply a *constant* either equal to $(\lambda+n)$ as in the Solow-Swan type models or to μn as in the multiples-of- n models. Rather, the optimum growth of the money stock is a variable that is sensitive to changes in all the structural parameters of the model. Once these parameters are fixed, then the optimum rate of growth of the money stock is determined. When these parameters change over time, so does the optimum growth rate of the quantity of money.

To illustrate the mechanism behind the productivity multiplier models, consider the simplest model:

$$(8) \quad Q = yN \quad (\text{Production Function})$$

$$(9) \quad \dot{y} = s(Q/N) - (\dot{M}/P) - \delta y \quad (\text{Technical Change Function})$$

$$(10) \quad (\dot{M}/P) = bg(Q/N) \quad (\text{Money Growth Equation})$$

$$(11) \quad \dot{N}/N = n \quad (\text{Population Growth Equation})$$

where the variables are defined as before and δ = rate of depreciation, $g = \dot{Q}/Q - n = \dot{y}/y$, and b = desired cash-income ratio.

Eqs. (8) through (11) reduce to

$$(12) \quad \dot{y}/y = (s-\delta)/(1+b) = \dot{M}/M - n$$

if and only if $\dot{P}/P = 0$. Note that, while the above simple productivity model is limited to steady growth, it nevertheless possesses some characteristics more useful than those of the previous models: (i) the optimum growth rate of the money stock now depends on the savings rate s , desired cash-income ratio b , and depreciation rate δ ; (ii) no aggregate capital stock concept is necessary; and (iii) no *exogenous* technical change is required to explain a positive optimum per capita growth rate of the money stock.

2.3 Conlisk Model

The Conlisk model [2] differs from the simplest productivity multiplier model in one respect: the optimum per capita growth rate of the money stock in the Conlisk model depends on all structural parameters inclusive of the population growth rate and the parameters of the production function. The Conlisk model implies an inverse relationship between per capita growth rate of output (hence, optimum per capita growth rate of the money stock) and the population growth rate. Consider the following model:

$$(13) \quad Q = Q(K, L) \quad (\text{Production Function})$$

$$(14) \quad \dot{K} = sQ - (\dot{M}/P) - \delta K \quad (\text{Capital Growth Equation})$$

$$(15) \quad (\dot{M}/P) = bgQ \quad (\text{Money Growth Equation})$$

$$(16) \quad \dot{L} = hQ + nL \quad (\text{Labor Growth Equation})$$

where the variables are defined as before except for the following:

$$L = yN, \quad g = \dot{Q}/Q.$$

Eqs. (13) through (16) reduce to

$$(17) \quad \dot{k}/k = \{[s - bn - bhF(k,1)]F(1,1/k) - hF(k,1) - (n + \delta)\} / [1 + b\pi(k)F(1,1/k)]$$

where $k = K/L$, and $\pi(k) = kF_1(k,1)/F(k,1) =$ marginal productivity share of capital. If $b = 0$, Eq. (17) reduces to the basic (nonmonetary) Conlisk model. I have demonstrated in a paper on the Conlisk model [20] that if F is Cobb-Douglas, the slope and intersection conditions imply a well-behaved behavior for \dot{k}/k . Likewise, I have shown in the same paper that the optimum growth rate of the money stock is equal to $hAk^{*\alpha} + n$, where $\alpha = \pi(k)$ and A the constant term. The optimum per capita growth rate of the quantity of money is equal to $hAk^{*\alpha}$. Since k^* is itself a function of the structural parameters of the model, it follows that the optimum growth rate and per capita growth rate of the money stock are functions of the same parameters.

III. Summary and Conclusion

In summary, the following **sensitivities** for the optimum per capita growth rate of the quantity of money are

given for the various models discussed in this paper.

	<u>$\dot{(M/M-n)}^*$</u>			
	s	b	n	δ
Solow-Swan, Variable Bias	0	0	0	0
Multiples-of-n	0	0	+	0
Productivity Multiplier	+	-	0	-
Conlisk	+	-	-	-

Most modern discussions of the optimum growth rate of the quantity of money seem to base the optimum rate on the sum of the exogenous rate of Harrod-neutral technical change and the growth rate of population. This paper has attempted to show that this is a rather very limited and narrow view. The optimum growth rate and optimum per capita growth rate of the quantity of money depend on assumptions regarding the nature of technical change, i.e., whether or not technical change is exogenous. The differences are critical for monetary policy. If technical change is assumed to be exogenous, then the optimum growth rate of the quantity of money is $(\lambda+n)$ where λ = exogenous rate of Harrod-neutral technical change, regardless of changes in the structural parameters of the economic system. If technical change is assumed to be endogenous, then the optimum growth rate of the quantity of money is itself endogenously determined, i.e., when

structural parameters change, technical change adjusts appropriately, and so does the optimum growth rate of the money stock. It is true that in either type of models discussed in this paper there is a determinate optimum rate at which the money stock must grow. But it is only by a strange coincidence that we should expect the optimum rate deduced from Solow-Swan type models to be exactly equal to the optimum rate deduced from models where technical change is treated endogenously. Thus, the optimum growth rate deduced from the first category of models is a special case of the more general optimum growth rate expression deduced from the second class models. This special case emerges when $h = 0$, or when there exists no endogenous technical change.

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