the samples in each bracket were independently obtained, an estimate of variance of $\hat{p}$ turns out to be

$$(2) \qquad v(\hat{p}) = \sum_{h=1}^{10} \frac{N_h^2}{N^2} \cdot \frac{N_h - n_h}{N_h} \cdot \frac{\hat{p}_h(1 - \hat{p}_h)}{n_h - 1}$$

(1) is found to be .6004 with estimated variance (2) computed at .000005479. It may be noted that (1) and (2) are unbiased estimators of their respective population values if the Manila list is a stratified simple random sample of the total 1960 taxable returns in the Philippines.

2.2A Estimating the number or proportion of married taxable returns in sub-brackets

Since the sample sizes of single taxable returns and of married taxable returns are each pre-determined, the number or proportion of married taxable returns in the sub-brackets of the total 1960 Philippine list cannot be estimated using an estimator analogous to (1). An estimator $\hat{N}'_m$, however, can be devised utilizing the per cent distribution of married taxable returns (or of single taxable returns if we use an estimator $\hat{N}'_s$, the subscript s is for single) both in sub-brackets of Table 2.3A and in brackets of Table 2.2, column (5) (or column (3) for single).

Define the following symbols in the Philippine list by:

N_m = number of married taxable returns

p_m = proportion of married taxable returns

N_{mh} = number of married taxable returns in the hth bracket

δ_{mh} = per cent distribution of the hth bracket among the married taxable returns

N_{mhi} = number of married taxable returns in the ith subbracket of the hth bracket

δ_{mhi} = per cent distribution of the ith subbracket in the hth bracket of the married taxable returns.

The analogous symbols for the properties in the Manila list and the subsamples are indicated by a circumflex "^" above the symbols. From the definitions $\sum_i \delta_{mhi}$ and $\sum_h \delta_{mh}$ are each equal to unity. Similarly, $\sum_{i,h} N_{mhi} = \sum_h N_{mh} = N_m$.

Since $\delta_{mh} = N_{mh}/N_m$, $\delta_{mhi} = N_{mhi}/N_{mh}$, and $p_m = N_m/N$

(1) $p_m \cdot \delta_{mh} \cdot \delta_{mhi} = N_m/N \cdot N_{mh}/N_m \cdot N_{mhi}/N_{mh}$

$= N_{mhi}/N = \delta'_{mhi}$, per cent distribution of the ith subbracket in the hth bracket among the total list of 1960 taxable returns.

From the last set of equality in (1), $N_{mhi} = N\delta'_{mhi}$. An estimate of N_{mhi} from the sample may be given by

(2) $\hat{N}_{mhi} = N\hat{\delta}'_{mhi} = N\hat{p}_m \cdot \hat{\delta}_{mh} \hat{\delta}_{mhi}$

Section 2.2 gives $\hat{p}_m = \hat{p} = .6004$. For the second subbracket of the first bracket, $\hat{\delta}_{m1} = .4411$ (Table 2.2, column (5)) and $\hat{\delta}_{m12} = .4233$ (Table 2.3A, under column "married").

From (2) above,

$$\hat{\delta}'_{mhi} = \hat{N}_{mhi}/N = \hat{p}_m \cdot \hat{\delta}_{mh} \hat{\delta}_{mhi}$$

$$= (.6004)(.4411)(.4233) = .1121$$

The "1-2" thousand subbracket of the married group amounts to about 11.21% of the total number of taxable returns in 1960. Using (2) above, an estimated number runs to 103,137 x .1121 = 11,562 married taxable returns in 1960.

To find the variance of $\hat{N}_{mhi}$, we have

(3) $$V(\hat{N}_{mhi}) = E[\hat{N}_{mhi} - E(\hat{N}_{mhi})]^2$$

$$= E(\hat{N}_{mhi})^2 - \{E(\hat{N}_{mhi})\}^2$$

(4) $$E(\hat{N}_{mhi}) = E_3 E_2 E_1 (\hat{p}_m \hat{\delta}_{mh} \hat{\delta}_{mhi}),$$

where E_1 refers to conditional expectation over i for fixed h among married taxable returns,

E_2 refers to conditional expectation over h among married taxable returns, and

E_3 refers to expectation of proportion of married taxable returns.

Thus,

(5) $$E(\hat{N}_{mhi}) = E_3[\hat{p}_m \cdot E_2\{\delta_{mh} \cdot E_1(\delta_{mhi})\}]$$

$$= E_3[\hat{p}_m \cdot E_2\{\delta_{mh} \cdot \bar{\delta}_{mh}\}]$$

$$= Np_m \text{ cov } (\delta_{mh}, \bar{\delta}_{mh})$$

(6) $$E(\hat{N}_{mhi})^2 = N^2 E_3[\hat{p}^2_m \cdot E_2\{\delta^2_{mh} \cdot E_1(\delta^2_{mhi})\}]$$

Combining (5) and (6), we obtain an expression for the variance (3). An estimate of (3) can be obtained from the sample by a tedious computational scheme.

2.3 The Projection of Individual Income Tax

The general procedure having been followed was along the same lines as that in Part I; namely, to have an estimate of net taxable income by bracket and an estimate of number of taxable returns for the same bracket. However, it is believed that estimation in this sector has been more difficult than the corporation sector since: (i) the number of brackets involved are more numerous (23 brackets against 4 previously), (ii) certain changes in personal deductions left us only with years 1959 through 1962 which have similar historical exemptions, (iii) the dearth of data on percentage distribution by bracket led us to utilize the percent distribution of the 1960 list for separating the number of taxable returns into taxable brackets.

(A) If we assume that the list (Table 2.1) of 1960 taxable individual returns is a random sample from an infinite population (that is, the 1960 list is a sample from what could have been all possible lists through time) then the means of taxable assessments for each bracket in the 1960 list can be utilized as estimates of the population means. The population variance and, hence, the population standard deviation may be estimated from the sample of size 1,073 previously obtained. Assuming now the sample variances to be approximately those of the corresponding population values, by the Central Limit Theorem, the hth bracket mean of the 1960 list approaches that of the normal distribution with the same mean and variance equal to $1/n_h$ of the population values, where n_h is the sample size of the hth stratum.

With the 1960 means in each bracket used as the middle estimate, low and high estimates similarly can be obtained from the fiducial limits of the 95% confidence interval estimates. These estimates of the bracket means are tabulated under Table 2.4.

(B) The growth of individual taxable returns from 1959 (t=0) through 1962 were:

t:	0	1	2	3
R_t:	90,364	103,337	118,844	137,935

Transforming R_t into logarithms (base 10) and fitting by least squares gives

$$\log_{10} R_t = 0.78576 + 0.04917t \qquad (3)$$

with estimated standard deviation $\hat{\sigma}_{\log_{10} R}$ of $\log_{10} R$, given by

$$\hat{\sigma}_{\log_{10} R} = .01758. \qquad (4)$$

Projections to 1963 and 1964 are obtained by putting t=4 and t=5, respectively, in equation (3). Thus, after taking the antilogarithms of the projected $\log_{10} R_t$, we have

	1963(t=4)	1964(t=5)
R_t:	151,028	169,136

Percentage distribution of the 1960 list, when applied to the projected values for 1963 and 1964, respectively, gives results in Table 2.5 below.

Table 2.4

INTERVAL ESTIMATES OF MEAN NET INCOME BY TAX BRACKET, FOR PRESENT BRACKET SIZES

(Pesos)

Present Tax Brackets	Mean Net Income Low	Mean Net Income Middle	Mean Net Income High
Zero- 2,000	627	697	767
2,001- 4,000	2,867	2,969	3,071
4,001- 6,000	5,497	5,633	5,769
6,001- 8,000	7,321	7,464	7,607
8,001- 10,000	9,524	9,699	9,874
10,001- 20,000	13,063	13,999	14,935
20,001- 30,000	23,893	24,784	25,675
30,001- 40,000	33,660	34,566	35,472
40,001- 50,000	45,854	46,689	47,524
50,001- 60,000	54,048	54,860	55,672
60,001- 70,000	65,805	66,766	67,727
70,001- 80,000	76,241	77,010	77,779
80,001- 90,000	84,115	85,152	86,189
90,001-100,000	92,942	94,048	95,154
100,001-120,000	107,960	109,462	110,964
120,001-140,000	126,144	128,157	130,170
140,001-160,000	148,754	150,926	153,098
160,001-200,000	173,110	178,395	183,680
200,001-250,000	215,937	225,667	235,397
250,001-300,000	260,331	268,949	277,567
300,001-400,000	331,395	355,375	379,355
400,001-500,000	402,540	448,863	495,186
500,001 and over	855,882	955,667	1,055,452

Table 2.5

Projected Number $R_t^{(h)}$ [1] of Individual Taxable Returns, for 1964 and 1965 and Estimates of the Mean Accumulated Taxes a_h

Bracket (Thousand Pesos)	Per Cent Distribution	$R_4^{(h)}$ 1963	$R_5^{(h)}$ 1964	a_h Low	a_h Middle	a_h High
0 - 2	63.58	96,042	107,555	18.81	20.91	23.01
2 - 4	15.421	23,293	26,086	112.02	118.14	124.26
4 - 6	6.905	10,430	11,680	314.73	326.97	339.21
6 - 8	3.764	5,686	6,367	571.36	594.24	617.12
8 - 10	2.390	3,610	4,043	984.80	1,019.80	1,054.80
10 - 20	4.345	6,563	7,350	1,815.12	2,039.76	2,264.40
20 - 30	1.568	2,368	2,652	4,647.90	4,915.20	5,182.50
30 - 40	.726	1,097	1,228	7,797.60	8,123.76	8,449.92
40 - 50	.426	643	721	12,421.60	12,755.60	13,089.60
50 - 60	.263	379	445	15,780.16	16,121.20	16,462.24
60 - 70	.140	211	237	20,834.20	21,257.04	21,678.88
70 - 80	.091	137	154	25,550.86	25,904.60	26,258.34
80 - 90	.063	95	107	29,255.20	29,752.96	30,250.72
90 -100	.061	92	103	33,551.00	34,104.00	34,657.00
100-120	.077	116	130	41,219.20	42,000.24	42,781.28
120-140	.049	74	83	50,736.32	51,803.21	52,870.10
140-160	.026	39	44	62,807.16	63,980.04	65,152.92
160-200	.036	54	61	76,090.50	78,997.25	81,904.00
200-250	.017	26	29	99,804.72	105,253.52	110,702.32
250-300	.012	18	20	124,768.67	129,680.93	134,593.19
300-400	.007	11	12	165,589.10	179,497.50	193.405.90
400-500	.006	9	10	206,878.60	234,209.17	261.539.74
Above 500	.011	17	19	477,909.20	537,780.20	597,651.20
	100.000	151,028	169,136			

[1] The projection of R_t is somewhat higher than the projection using simple regression without transforming R_t into its logarithm.

(C) Denote by $\bar{x}_h$ the mean taxable assessment for the hth bracket. If r_h is the marginal rate for the hth bracket, an estimate of the mean accumulated tax a_h is

$$(5) \qquad a_h = \sum_{j=1}^{h-1} r_j (b_{j2} - b_{j1}) + r_h(\bar{x}_h - b_{h-1})$$

where b_{j1} and b_{j2} are, respectively, the "lower" and "upper" boundaries of the jth bracket. Total tax assessment (middle) of the hth bracket may now be estimated by multiplying the projected number $R_t^{(h)}$ in year t of the hth bracket obtained from Table 2.5 with equation (5) above.

Since the variance of a_h is

$$(6) \qquad V(a_h) = r_h^2 \, V(\bar{x}_h),$$

the standard error of a_h may be computed from low and high estimates of $\bar{x}_h$ given in Table 2.4 by simply multiplying 1/2 the difference of high and low estimates by the corresponding r_h. Using (5) and the standard error of a_h, low and high estimates of a_h can be computed from the 95% confidence limits for a_h. These are also given in Table 2.5. From (B) and (C) above, Table 2.5 may now be utilized to obtain the projected low, middle and high individual tax assessment by multiplying $R_t^{(h)}$ by the corresponding low, middle and high estimates for a_h. The results are tabulated (Table 2.6) below:

Table 2.6

PROJECTIONS OF TOTAL INDIVIDUAL TAX ASSESSMENTS
BY BRACKETS: 1963-1964
(Pesos)

Brackets	1963 Low	1963 Middle	1963 High	1964 Low	1964 Middle	1964 High
0 - 2,000	1,806,550	2,008,238	2,209,926	2,023,110	2,248,975	2,474,841
2,001 - 4,000	2,609,282	2,751,835	2,894,388	2,922,154	3,081,800	3,241,
4,001 - 6,000	3,282,634	3,410,297	3,537,960	3,676,046	3,819,010	3,961,973
6,001 - 8,000	3,248,753	3,378,849	3,508,944	3,637,849	3,783,526	3,929,203
8,001 - 10,000	3,555,128	3,681,478	3,806,745	3,981,546	4,123,051	4,264,556
10,001 - 20,000	11,912,633	13,386,945	14,861,257	13,341,132	14,992,236	16,643,340
0,001 - 30,000	11,006,227	11,639,194	12,272,160	12,326,231	13,035,110	13,743,990
0,001 - 40,000	8,553,967	8,911,765	9,269,562	9,575,453	9,975,977	10,376,502
0,001 - 50,000	7,987,089	8,201,851	8,416,613	8,955,974	9,196,788	9,437,602
0,001 - 60,000	6,264,724	6,400,116	6,535,509	7,022,171	7,173,934	7,325,697
0,001 - 70,000	4,396,016	4,485,235	4,574,244	4,937,705	5,037,918	5,137,895
0,001 - 80,000	3,500,468	3,548,930	3,597,393	3,934,832	3,989,308	4,043,784
0,001 - 90,000	2,779,244	2,826,531	2,873,818	3,130,306	3,183,567	3,236,827
,001 - 100,000	3,086,692	3,137,568	3,188,444	3,455,753	3,512,712	3,569,671
,001 - 120,000	4,781,427	4,872,028	4,962,628	5,358,496	5,460,031	5,561,566
,001 - 140,000	3,754,488	3,833,438	3,912,387	4,211,115	4,299,666	4,388,218
,001 - 160,000	2,449,479	2,495,222	2,540,964	2,763,515	2,815,122	2,866,728
,001 - 200,000	4,108,887	4,265,852	4,422,816	4,641,521	4,818,832	4,996,144
,001 - 250,000	2,594,923	2,736,592	2,878,260	2,894,337	3,052,352	3,210,367
,001 - 300,000	2,245,836	2,334,257	2,422,677	2,495,373	2,593,619	2,691,864
,001 - 400,000	1,821,480	1,974,473	2,127,465	1,987,069	2,153,970	2,320,871
,001 - 500,000	1,861,907	2,107,883	2,353,858	2,068,786	2,342,092	2,615,397
... 500,000	8,124,456	9,142,263	10,160,070	9,080,275	10,217,824	11,355,373
	105,732,290	111,530,840	117,328,088	118,420,749	124,907,420	131,393,855

2.4 Mean Taxable Income and Percent Distribution of Smaller Brackets

The sub-sample from the Manila list may be utilized to obtain the breakdown into smaller brackets. The proportions of smaller brackets in the larger brackets have been derived from the sample together with the mean taxable income in each smaller bracket. If the marginal rates are known for each of these sub-brackets average cumulated tax assessments may be made for each of the sub-brackets just as in Table 2.5. Subdividing the projected number of returns of the larger brackets by the known proportion of the smaller brackets (Table 2.3A) and multiplying these numbers by the average cumulated tax assessments will give us again the required tax assessments by smaller brackets.

Table 2.7 below shows the mean taxable incomes in the sub-brackets obtained from the 1,073 sub-sample of the Manila list. It may be remarked that, although proportions of married (or single) taxable returns cannot be used by pooling sub-samples of single and married groups (first paragraph of section 2.2A), the mean taxable incomes of sub-brackets may be pooled from the means of the sub-bracket in the single and married groups. Denote by $\bar{x}_1$ the sample mean of single taxable return in a sub-bracket and $\bar{x}_2$ the sample mean of married taxable return in the same sub-bracket. The mean taxable return $\bar{x}$ for the particular sub-bracket is

$$\bar{x} = \frac{n_1'\bar{x}_1 + n_2\bar{x}_2}{n_1' + n_2'},$$

Table 2.7 PROPORTION AND MEAN TAXABLE INCOMES BY SMALLER BRACKETS

Bracket (₱1,000)	Sub-bracket (₱1,000)	Proportion	Mean Taxable Income (₱1,000)
0 - 2		1.00	
	0 - 1	.6537	.4524
	1 - 2	.3463	1.4944
2 - 4		1.00	
	2 - 3	.6207	2.4444
	3 - 4	.3793	3.4545
4 - 6		1.00	
	4 - 5	.5152	4.5882
	5 - 6	.4848	5.5000
6 - 8		1.00	
	6 - 7	.5636	6.5806
	7 - 8	.4364	7.5000
8 - 10		1.00	
	8 - 9	.6667	8.5357
	9 - 10	.3333	9.5714
10 - 20		1.00	
	10 - 12	.3846	10.8000
	12 - 14	.2051	13.0000
	14 - 16	.2051	15.0000
	16 - 18	.0769	17.6667
	18 - 20	.1282	19.0000
20 - 30		1.00	
	20 - 22	.2195	20.7778
	22 - 24	.1951	23.1250
	24 - 26	.2195	24.8889
	26 - 28	.1951	26.7500
	28 - 30	.1707	29.0000
30 - 40		1.00	
	30 - 32	.2903	31.3000
	32 - 34	.16215	32.8333
	34 - 36	.2703	34.8000
	36 - 38	.1351	37.2000
	38 - 40	.16215	38.8333

Bracket (₱1,000)	Sub-bracket (₱1,000)	Proportion	Mean Taxable Income (₱1,000)
40 - 50		1.00	
	40 - 42	.2000	41.1111
	42 - 44	.1333	43.1667
	44 - 46	.2667	45.0000
	46 - 48	.1556	46.7143
	48 - 50	.2444	48.9091
50 - 60		1.00	
	50 - 52	.2759	50.8125
	52 - 54	.2069	53.0833
	54 - 56	.1552	54.7778
	56 - 58	.1379	56.8750
	58 - 60	.2241	59.0000
60 - 70		1.00	
	60 - 62	.2558	60.6360
	62 - 64	.2093	63.0000
	64 - 66	.1395	64.6670
	66 - 68	.1860	67.1250
	68 - 70	.2093	69.0000
70 - 80		1.00	
	70 - 75	.5079	72.4370
	75 - 80	.4921	77.8710
80 - 90		1.00	
	80 - 85	.4848	82.4370
	85 - 90	.5152	87.9410
90 -100		1.00	
	90 - 95	.5556	92.1330
	95 -100	.4444	97.2500
100-120		1.00	
	100-105	.3400	102.3530
	105-110	.2600	107.7690
	110-120	.4000	114.0500
120-140		1.00	
	120-130	.6333	125.9470
	130-140	.3667	135.7270

Bracket (₱1,000)	Sub-bracket (₱1,000)	Proportion	Mean Taxable Income (₱1,000)
140-160		1.00	
	140-145	.5833	142.1430
	145-150	.2500	147.3330
	150-155	.1670	151.0000
	155-160	. 0	
160-200		1.00	
	160-170	.3333	165.0000
	170-180	.3333	175.1429
	180-190	.1111	185.0000
	190-200	.2223	196.4000
200-250		1.00	
	200-210	.2500	204.6670
	210-220	.0833	214.0000
	220-230	.0833	226.0000
	230-240	.2500	236.3333
	240-250	.3300	245.0000
250-300		1.00	
	250-260	.3636	253.2500
	260-270	.0909	265.0000
	270-280	.2727	274.3333
	280-290	.1818	281.0000
	290-300	.0909	296.0000
300-400		1.00	
	300-350	.800	319.0000
	350-400	.200	376.0000
400-500		1.00	
	400-450	.6667	425.5000
	450-500	.3333	496.0000
Above 500		1.00	907.143

where n_1' and n_2' are, respectively, the sample sizes of single and married groups in the same sub-bracket.

2.5 Projections Under Proposed Brackets (PIA).

Following the brackets proposed by PIA, individual tax assessments for 1963 and 1964 have been worked out (Table 2.8). The number of returns in the proposed brackets were each derived from the sample proportions shown in Table 2.7 with some adjustments if the proposed ones have boundaries other than those obtainable in that table. The mean cumulated tax assessments were similarly derived as in section 2.3 (equation 5) under the proposed marginal rates. The mean taxable incomes were obtained from the sample data. Compared with Table 2.6 the middle estimates for the corresponding years and the projections total under the PIA proposal differ by about ₱11 million.

2.6 The Projection of Total Individual Income Tax

The projections in section 2.3 of the individual income tax assessments by brackets for the years 1963 and 1964 are cross-sectional in nature. The total of middle estimates are values resulting from an estimator of the total individual tax assessments. For some fixed number of individual taxable returns $R_t^{(h)}$ in year t of the h^{th} bracket the tax assessment is $R_t^{(h)} \times a_h$, where a_h is now the average accumulated tax assessment in the h^{th} bracket. Since a_h (from (5) of that section) is a linear function of $\bar{x}_h$ (mean taxable income in the h^{th} bracket) and $\bar{x}_h$ in large