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The Robust Nash Equilibrium and Equilibrium Selection in 2x2 Coordination Games

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Abstract

We propose an equilibrium concept, the Robust Nash equilibrium (RNE), that combines the best-reply rationality and the “first mover invariance” condition. The single-stage 2x2 symmetric information game G is transformed into sequential two-stage games with two subtrees: ST_A has the row player starting and ST_B has the column player starting. A profile in G is robust if it is the strict SPNE of the two branches; it is ephemeral if it is not the SPNE of any branch. We show that every strict dominant strategy equilibrium of G is robust but not every strict Nash equilibrium of G is. We show further that every robust profile of G is always a strict Nash equilibrium of G . A Robust Nash equilibrium (RNE) of G is any robust profile of G . The RNE of G is unique. We show in particular that the payoff dominant strict Nash equilibrium of a coordination game G is RNE while the strictly payoff-dominated Nash equilibrium of G is ephemeral. The original Harsanyi-Selten preference for payoff dominance over risk dominance is supported by robustness without invoking collective rationality. JEL Classification: C02, C72.

I. Introduction

Simple strategic games have sometimes too many Nash equilibria and sometimes none. Adding plausible conditions on top of the best-reply rationality is one way to reduce the number of equilibria. Richard Selten’s sub-game perfect Nash equilibrium, which combines the best-reply rationality of Nash and the backward induction rationality allows the rejection of some Nash equilibria. A. Guth’s Ultimatum Game has 101 Nash equilibria but has a unique sub-game perfect equilibrium. When randomization is combined with best-reply rationality, the mixed strategy Nash equilibrium always exists where a Nash equilibrium in pure strategies does not. The 2x2 Coordination Game has two strict Nash equilibria and the problem of equilibrium selection arises. In 1988, Harsanyi and Selten proposed two concepts to guide equilibrium selection in coordination games: payoff dominance and risk dominance. A Nash equilibrium profile payoff-dominates another if it gives every player a strictly larger payoff than the other. It risk-dominates another if the cost of deviations from the former exceeds the cost of deviations from the latter. Harsanyi and Selten argued from the viewpoint of collective rationality that payoff dominance should be granted primacy over risk dominance whose claim hangs on individual rationality. Carlsson and Van Damme (1992) have shown that in global games where players only observe noisy signals of the payoffs and the iterative dominance condition is applied, the equilibrium that emerges in the limit as the noise approaches zero is unique. This limit uniqueness theorem has been extended by Carlsson and Van Damme (1993) and Kim (1996) and finally generalized by Frankel, Morris and Pauzner (2002). In a 2x2 coordination game with full information, this unique profile is the risk-dominant profile. Thus, risk dominance appears to have gained favor in global games of noisy signals, individually rational

agents and strategic complementarities. Another area where risk dominance received support is in evolutionary games. In replicator dynamics models, the likelihood that the population will choose the risk dominant profile is higher (see, e.g., Samuelson, 1997). In best response strategy revision with mutation, as the probability of mutation approaches zero, the choice of the risk dominant profile is a certainty (Kandori, Mailath, Rob, 1993; Kim, 1993). Harsanyi himself (1995) switched to favoring risk dominance. But the victory has not been clinched. One should not be surprised that models based on individual rationality should privilege risk-dominance. Camerer (2003) would still observe that “predicting which of the many equilibria will be selected is perhaps the most difficult problem in game theory.”

Numerous and increasing laboratory and field experiments seem to show (see, e.g., Ostrom 2000) that small groups often succeed in reaching and persisting in payoff-superior solutions to the common resource problem in defiance of the “tragedy of the commons.” In other words, the social context in which agent decisions are being made is emerging as a strong explanatory idea. Ostrom and her school (1990, 2000, 2009) have identified some of the group characteristics that make this outcome likely (small number, face-to-face repeated/sequential interaction, salience of the collective success, relative heterogeneity of membership, difficulty of exit and penalty for norm violation). Ostrom herself favored conditional cooperation in place of rational egoism as explanation for these successes (see, e.g., Ostrom, 2009). Ideas like reciprocity, group identity, group norms and social capital making for greater trust and social coherence, in other words, the social context of the interactions, (see e.g., Knack and Keefer, 1997; Akerlof and Kranton, 2000; 2010; Ostrom, 2009; Fehr, 2009) have been proposed in the empirical and theoretical literature as possible candidate explanations for observed heterodox phenomena. These support the Harsanyi-Selten predilection for collective rationality and, thus, payoff dominance. Risk dominance crucially hangs on one agent’s uncertainty about how the other agent will behave.

In this paper we will take the sequential and face-to-face nature of the interaction in small groups as essential. In Section II, we introduce the transformation of an arbitrary 2x2 single stage full information game G into sequential full information two-stage games with two sub-trees, one for each player as first mover. We then introduce the concept of a robust profile of G : one that emerges as the strict SPNE of the two branches. A game having a robust profile satisfies the “first mover invariance” condition. Profiles that do not emerge at all in any branch is called ephemeral. We then prove various relationships of robust profiles to dominant strategy and Nash equilibrium concepts, in particular, that every dominant strategy equilibrium of G is robust; that every robust profile of G is a strict Nash equilibrium, but not vice versa. A profile is a Robust Nash equilibrium of G if it is a robust profile of G . The Robust Nash equilibrium of the 2x2 G is unique. In Section III, we apply the results to equilibrium selection in coordination games. We show that every payoff-dominant Nash equilibrium of coordination game G is a robust Nash equilibrium while the risk-dominant Nash equilibrium is an ephemeral Nash equilibrium. The robust Nash equilibrium concept, thus, privileges payoff dominance in coordination games. We then conclude.

II. Stage Transformation (ST)

Let G be a 2x2 single-stage full information game, as in **Table 1**, and let $\{H, \sim H\}$ be the strategy set of A and $\{T, \sim T\}$ be the strategy set of player B .

Table 1. Single-stage full information game, G

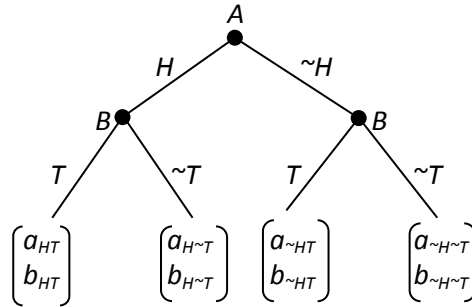
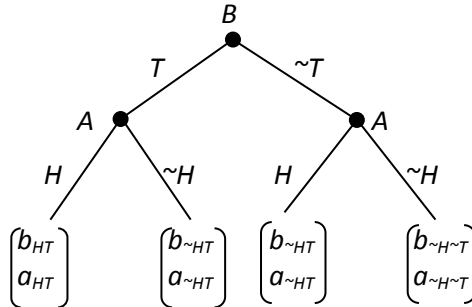
		Player B	
		T	$\sim T$
Player A	H	a_{HT}, b_{HT}	$a_{H\sim T}, b_{H\sim T}$
	$\sim H$	$a_{\sim HT}, b_{\sim HT}$	$a_{\sim H\sim T}, b_{\sim H\sim T}$

We first define a transformation of G from a single-stage game to a sequential 2-stage game to incorporate the backward induction rationality.

Definition 1: Let ST be the stage transformation of G into a 2x2, symmetric information two-stage game involving (i) two two-stage sub-trees: (a) ST_A : Row player A starts in stage 1 and column player B responds in stage 2; (b) ST_B : B starts in stage 1 and A responds in stage 2; (ii) the payoffs for each sequence of actions in a sub-tree are identical to the corresponding quadrant payoffs in G ;

Definition 2: The ST of G defined in Definition 1 we label G' . The equilibrium concept applying in G' is the “sub-game perfect Nash equilibrium” (SPNE).

The two sub-trees of ST , denoted ST_A when player A starts and ST_B when player B starts, are given in **Figures 1** and **2**, respectively.

Figure 1: ST_A (A Starts)**Figure 2:** ST_B (B Starts)

Note: The payoffs for each sequence are identical to corresponding quadrant payoffs of G .

Definition 3: A profile (h, t) of G where $h = H, \sim H$, and $t = T, \sim T$ is *robust* if it is the strict SPNE of the two sub-trees of G' ; (ii) it is *ephemeral* if it fails to be the strict SPNE of any sub-tree of G' .

Remark 1: By “strict SPNE” we mean that at no stage is any player indifferent between his/her options.

Remark 2: Robustness is equivalent to the condition “first mover invariance.” Under this condition the first mover advantage does not arise. A profile of G may emerge as the strict SPNE of one sub-tree of G' but not of the other. We will not deal with this property in this paper.

While the equilibrium concept used in G can be any accepted concept satisfying the best-reply rationality (e.g., Nash, or Dominant Strategy), the equilibrium concept employed in G' embodies the backward induction rationality, that is, the “sub-game perfect Nash equilibrium” (SPNE). A robust profile of G makes it special in the sense that when the game G is transformed into G' , the “first mover advantage” is effectively suppressed. Thus, the robustness condition is equivalent to a “no first mover advantage condition.” We now enquire about the robustness property of certain interesting equilibrium profiles of G .

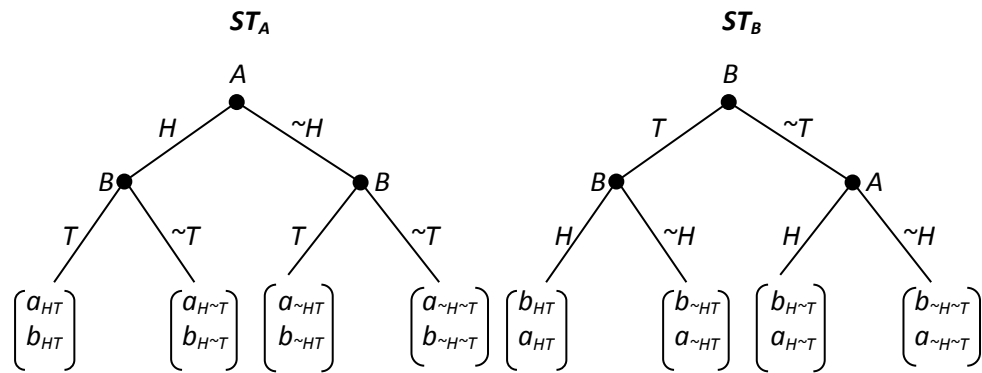
Remark: In this paper we will restrict attention to strict equilibrium profiles.

The strict dominant strategy equilibrium profile is of some interest.

Proposition 1: Suppose a profile (h, t) of G is the strict dominant strategy equilibrium profile of G . Then (h, t) is a robust profile of G .

Proof: Let $(h, t) = (H, T)$ without loss of generality. The two sub-trees of ST are as in **Figure 3**:

Figure 3. STs of G' .



A strategy profile (H, T) is a strict dominant strategy equilibrium of G if H strictly dominates $\sim H$ for A and T strictly dominates $\sim T$ for B, that is, $a_{HT} > a_{\sim H T}$, $a_{H\sim T} > a_{\sim H \sim T}$ and $b_{HT} > b_{H\sim T}$, $b_{\sim H T} > b_{\sim H \sim T}$. In stage 1 of ST_A of G' , since $b_{HT} > b_{H\sim T}$ and $b_{\sim H T} > b_{\sim H \sim T}$, A chooses between H giving a_{HT} and $\sim H$ giving $a_{\sim H T}$ in the reduced form. Since $a_{HT} > a_{\sim H T}$ by (H, T) being a strict DSE, A chooses H in stage 1. B's best reply to H in stage 2 is T . So (H, T) is strict SPNE of ST_A .

In ST_B stage 1, since $a_{HT} > a_{\sim HT}$ and $a_{H\sim T} > a_{\sim H\sim T}$, B chooses between T giving b_{HT} and $\sim T$ giving $b_{H\sim T}$. B chooses T since $b_{HT} > b_{H\sim T}$. A 's best reply to T in stage 2 is H . Thus (H, T) is strict SPNE of ST_B . Thus (H, T) is a robust profile of G . The proof for $(h, t) = (H, \sim T)$ or $(\sim H, T)$ or $(\sim H, \sim T)$ is analogous. QED

Example 1: Consider a game G where (C, D) is the strategy set for both players A and B . Let (C, C) be the strict dominant strategy equilibrium of G . Then $a_{CC} > a_{DC}$ and $a_{CD} > a_{DD}$ and $b_{CC} > b_{CD}$ and $b_{DC} > b_{DD}$, that is, C strictly dominates D for A and C strictly dominates D for B . In ST_A of G , stage 1, since $b_{CC} > b_{CD}$ and $b_{CD} > b_{DD}$, A chooses between C giving a_{CC} and D giving a_{DC} in the reduced form. Since $a_{CC} > a_{DC}$ by (C, C) being a strict NE, A chooses C in stage 1. B 's best reply to C in stage 2 is C . So (C, C) is strict SPNE of ST_A . In ST_B stage 1, since $a_{CC} > a_{DC}$ and $a_{CD} > a_{DD}$, B chooses between C giving b_{CC} and D giving b_{CD} . B chooses C since $b_{CC} > b_{CD}$. A 's best reply to C in stage 2 is C . Thus (C, C) is strict SPNE of ST_B . Thus (C, C) is a robust profile of G .

By contrast, we show that a strict Nash equilibrium of G need not be robust.

Example 2: Consider the game G given in **Table 2**.

Table 2. Single-stage full information game, G .

		Player B	
		C	D
Player A	C	8, 9	3, 8
	D	10, 3	5, 5

where (C, D) is the strategy set for both players A and B . (D, D) is a unique strict Nash equilibrium of G . In ST_A stage 1, the reduced form has A choosing between C giving 8, and D giving 5 and A chooses C . In stage 2, B 's best-reply to C is C , since $9 > 8$. Thus, (C, C) is strict SPNE of ST_A . It is easy to see that (D, D) is the strict SPNE of sub-tree ST_B . Thus, (D, D) is not robust. We have shown the following:

Proposition 2: Not every strict Nash equilibrium profile of G is robust.

Proposition 3 spells out some conditions under which a strict Nash equilibrium is robust.

Proposition 3: Suppose $(h, t) = (H, T)$ is a strict Nash equilibrium of G . If either

- (i) $a_{H\sim T} > a_{\sim H\sim T}$ and $b_{\sim HT} > b_{H\sim T}$
- (ii) $a_{HT} > a_{\sim H\sim T}$, $a_{H\sim T} > a_{\sim H\sim T}$, and $b_{\sim H\sim T} > b_{\sim HT}$
- (iii) $a_{\sim H\sim T} > a_{H\sim T}$, $b_{HT} > b_{\sim H\sim T}$, and $b_{\sim HT} > b_{H\sim T}$
- (iv) $a_{HT} > a_{\sim H\sim T} > a_{H\sim T}$ and $b_{HT} > b_{\sim H\sim T} > b_{\sim HT}$

then (H, T) is a robust profile of G .

Proof: Confer **Figure 3** for this proof. (H, T) being a strict Nash equilibrium of G implies that $a_{HT} > a_{\sim HT}$ and $b_{HT} > b_{H\sim T}$. (i) If additionally condition (i) holds, then (H, T) is a dominant strategy equilibrium, and by Proposition 1 it is robust. (ii) Suppose condition (ii) holds. In stage 1 of ST_A , since $b_{HT} > b_{H\sim T}$ and $b_{\sim H\sim T} > b_{\sim HT}$, A chooses between H giving a_{HT} and $\sim H$ giving $a_{\sim H\sim T}$. Since (ii) assumes $a_{HT} > a_{\sim H\sim T}$, A chooses H and B 's best reply to H is T . Thus (H, T) is an SPNE in ST_A . In ST_B , since $a_{HT} > a_{\sim HT}$ and $a_{H\sim T} > a_{\sim H\sim T}$,

B chooses between T giving b_{HT} and $\sim T$ giving $b_{H\sim T}$ in stage 1. By the SNE assumption, B chooses T and A 's best reply to T is H giving a_{HT} over $\sim H$ giving $a_{\sim HT}$. (H, T) is, therefore, SPNE in ST_B and, hence, is robust under assumption (ii). (iii) Suppose additionally that (iii) holds. In ST_A , the reduced form has A choosing H with payoff a_{HT} over $\sim H$ with payoff $a_{\sim HT}$. B 's best reply to H is T since $b_{HT} > b_{H\sim T}$, thus (H, T) is SPNE in ST_A . In ST_B , the reduced form gives B b_{HT} for strategy T and $b_{\sim H\sim T}$ for $\sim T$. B chooses T since $b_{HT} > b_{\sim H\sim T}$ by (iii), and A 's best reply to T is H since $a_{HT} > a_{\sim HT}$ by SNE. (H, T) is, therefore, SPNE in ST_B and is robust under (iii). (iv) Finally, assume additionally that condition (iv) holds. In ST_A , the reduced form has A choosing H giving a_{HT} over $\sim H$ giving $a_{\sim H\sim T}$ since $a_{HT} > a_{\sim H\sim T}$. In stage 2, B 's best response to H is T since $b_{HT} > b_{H\sim T}$ by SNE. (H, T) is SPNE in ST_A . In ST_B , B is faced with choosing T giving b_{HT} and $\sim T$ giving $b_{\sim H\sim T}$ in stage 1. Since $b_{HT} > b_{\sim H\sim T}$ by (iv), B chooses T and A 's best response to T is H since $b_{HT} > b_{\sim HT}$, making (H, T) SPNE in ST_B . Thus, (H, T) is robust under condition (iv). The proofs for $(h, t) = (H, \sim T)$ or $(\sim H, T)$ or $(\sim H, \sim T)$ are analogous. QED

Example 3: To illustrate Propositions 1 and 3.iv, consider a game G where (C, D) is the strategy set for both players A and B .

Table 2a. Game G with a DSE.

		Player B	
		C	D
Player A	C	10,10	8,8
	D	6,6	7,5

Note: (C, C) is a strict Nash equilibrium and a DSE if $(a_{CD} > a_{DD}$ and $b_{DC} > b_{DD})$.

(C, C) is a strict Nash equilibrium since $10 > 6$ and $10 > 8$. (C, C) is also a dominant strategy equilibrium, hence (C, C) is robust by Proposition 1. Consider ST_A . In stage 2 sub-tree 1, B chooses between C giving 10 and D giving 8 and B chooses C . In stage 2, sub-tree 2, B chooses between C giving 6 and D giving 5 and B chooses C . In stage 1 of ST_A , A chooses between C giving 10 and D giving 6 and A chooses C . B 's best reply to C in sub-tree 1 stage 2 is C . Thus, (C, C) is a strict SPNE of ST_A . Consider ST_B . In stage 2, sub-tree 1, A chooses between C giving 10 and D giving 6 and A chooses C . In stage 2 sub-tree 2, A chooses between C giving 8 and D giving 7 so A chooses C . Thus, in stage 1, ST_B , B chooses between C giving 10 and D giving 8 and B chooses C . A 's best reply to C in stage two is C . Thus, (C, C) is strict SPNE of both ST_A and ST_B . Thus, (C, C) is a robust profile of G .

G -type games that have a robust profile are quite rare. Most games will have either no robust profiles or have two different profiles emerging as strict SPNEs of different sub-trees of G' or have only one profile emerging as a strict SPNE of one sub-tree. In other words, few games G will exhibit first mover invariance. In that case the "first mover advantage" does not arise. One can easily check, for example, that the "Tossing Pennies Game" does not have a robust profile. The following result is the first principal result of this paper:

Proposition 4: Every robust profile (h, t) of G is a strict Nash equilibrium of G .

Proof: Consider an arbitrary profile $(h, t) = (H, T)$. Let (H, T) be robust. This means (H, T) is a strict SPNE in both sub-trees of G' . In ST_A , (H, T) is a strict SPNE. A chooses H in stage 1 and B 's best reply to H is T in stage 2, that is, B chooses T with payoff b_{HT} over $\sim T$ with payoff $b_{H\sim T}$. By the strictness of SPNE, $b_{HT} > b_{H\sim T}$. In ST_B , since (H, T) is

SPNE, B chooses T in stage 1 and in stage 2 A chooses H with payoff a_{HT} over $\sim H$ with payoff $a_{\sim HT}$. This implies $a_{HT} > a_{\sim HT}$ under strict SPNE. The conditions $b_{HT} > b_{H\sim T}$ and $a_{HT} > a_{\sim HT}$ derived from ST_A and ST_B are exactly the conditions for a strict Nash equilibrium. The proof for the $(h, t) = (H, \sim T), (\sim H, T), (T, T)$ is analogous. QED

Corollary: The set of robust profiles of G is a strict subset of the non-empty set of SNE of G .

Proof: Let $\{R\}$ be the set of all robust profiles and $\{SNE\}$ be the non-empty set of strict Nash equilibria of G . By Proposition 4, every robust profile is SNE, that is, $\{R\} \subseteq \{SNE\}$. By Proposition 2, not all SNE is robust, implying $\{SNE\} \not\subseteq \{R\}$. Therefore $\{SP\} \subset \{SNE\}$. QED

Remark: It is now clear why the Tossing Pennies Game has no robust profile: the set of Nash equilibria is empty. The game in **Table 2** follows this rule: the set of robust profiles is empty while the static game has a unique Nash equilibrium.

III. The Robust Nash Equilibrium Concept

We now use the robustness property to introduce the following refinement of the Nash equilibrium concept using Proposition 4:

Definition 6: (i) Profile (h, t) of G is a *Robust Nash Equilibrium (RNE)* of G if it is a robust profile of G ; (ii) Profile (h, t) of G is an *Ephemeral Nash Equilibrium (ENE)* of G if it is a strict NE of G but is ephemeral.

Remark: Definition (6.i) follows from Proposition 4: every robust profile of G is a strict Nash equilibrium.

The RNE concept combines the original Nash best reply rationality and the “first-mover invariance” condition to form a refinement that is stronger than the SNE. We can then use this to select the preferred equilibrium in 2x2 coordination games. The following claims follow from previous propositions:

Proposition 5: (i) Every strict dominant strategy equilibrium of G is a RNE of G . (ii) Suppose a profile (h, t) of G is a strict Nash Equilibrium of G . If further either $(a_{HT} > a_{\sim H\sim T}, a_{H\sim T} > a_{\sim H\sim T}, \text{ and } b_{\sim H\sim T} > b_{\sim HT})$ or $(a_{\sim H\sim T} > a_{H\sim T}, b_{HT} > b_{\sim H\sim T}, \text{ and } b_{\sim HT} > b_{\sim H\sim T})$, then (h, t) is a RNE of G .

Proof: (i) Obvious from Proposition 1 which proves that every DSE of G is robust. (ii) We need to prove that (h, t) is robust under the given conditions. This is obvious from Proposition 3(ii) and 3(iii). Thus (h, t) is a Robust Nash equilibrium. QED

Proposition 6: The RNE of every 2x2 game G is unique.

Proof: Suppose $(h, t) = (H, T)$ is an RNE of G . Since (H, T) is robust, it is the strict SPNE of the two sub-trees of G' . Thus, no other profile of G can emerge as the strict SPNE of the sub-trees of G' . Thus (H, T) is unique. The proofs for $(h, t) = (H, \sim T), (\sim H, \sim T)$ and $(\sim H, T)$ are analogous. QED

Definition 7: Game G is “first mover invariant” if it has a RNE.

IV. Coordination Games, Payoff Dominance and Robustness

The issue of equilibrium selection arises in multiple equilibria games. It is especially salient in 2x2 coordination games. We first define coordination games.

Definition 8: G is a 2x2 coordination game if either $\{(H, T), (\sim H, \sim T)\}$ or $\{(H, \sim T), (\sim H, T)\}$ are strict Nash equilibria of G .

The equilibrium selection problem for the coordination game is one of choosing between two strict Nash equilibria of G , say (H, T) and $(\sim H, \sim T)$. Selten and Harsanyi (1988) proposed two selection criteria for equilibrium selection in coordination games: *payoff dominance* and *risk dominance*. We now introduce them with the view of bringing to bear the RNE concept.

Definition 9: Consider two strict Nash equilibria of coordination game G , (H, T) and $(\sim H, \sim T)$. (i) (H, T) of G payoff dominates $(\sim H, \sim T)$ if $a_{HT} > a_{\sim H \sim T}$ and $b_{HT} > b_{\sim H \sim T}$. (ii) (H, T) risk dominates $(\sim H, \sim T)$ if the following holds: $(a_{H \sim T} - a_{\sim H \sim T})(b_{\sim H T} - b_{\sim H \sim T}) > (a_{\sim H T} - a_{HT})(b_{HT} - b_{HT})$.

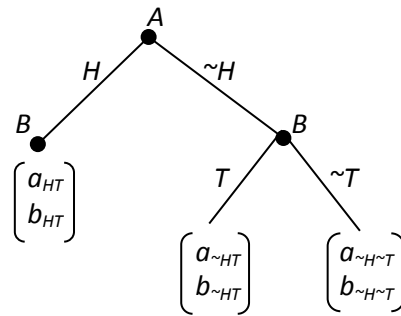
Remark: The expression on each side of the risk dominance inequality is the product of the deviation from each respective SNE; that is, (H, T) is risk dominant if the product of the costs of deviating from (H, T) exceeds that of deviating from $(\sim H, \sim T)$ in the first case. When the inequality holds, there is a presumption that (H, T) will be more adhered to than $(\sim H, \sim T)$ and, thus, be more stable. Or if mistakes are allowed, mistakes will be less likely in the case of (H, T) since the cost is more salient.

The following shows how the robustness property may imply payoff dominance.

Proposition 7: Let the profile $(h, t) = (H, T)$ be the RNE of G . If $a_{\sim HT} > a_{\sim H \sim T}$ and (ii) $b_{HT} > b_{\sim H \sim T}$, then the profile (H, T) payoff dominates $(\sim H, \sim T)$.

Proof: We wish to show that $a_{HT} > a_{\sim H \sim T}$ and $b_{HT} > b_{\sim H \sim T}$. In ST_A , (H, T) being robust implies that T will be chosen by B in stage 2 if A chooses H in stage 1. The reduced form for ST_A is shown in **Figure 6**.

Figure 6. Reduced form of ST_A of G' in Proposition 5.

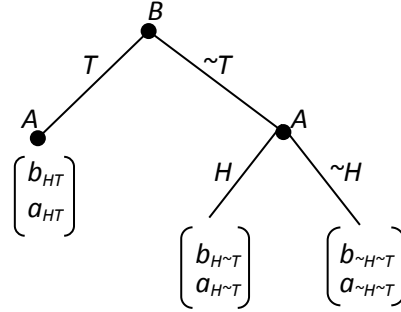


Player A will choose H over $\sim H$ only if a_{HT} is higher than the payoffs of $\sim H$. There are two cases: (a) if $b_{HT} > b_{\sim H \sim T}$, A will choose H if and only if $a_{HT} > a_{\sim HT}$. By assumption (i) and

transitivity, $a_{HT} > a_{\sim H \sim T}$. (b) if $b_{\sim HT} < b_{\sim H \sim T}$, then A will choose H over $\sim H$ if and only if $a_{HT} > a_{\sim H \sim T}$. In both cases $a_{HT} > a_{\sim H \sim T}$.

On the other hand, in ST_B , robustness of (H, T) implies that H will be chosen by A in stage 2 if B chooses T in stage 1. The reduced form is presented in **Figure 7**.

Figure 7. Reduced form of ST_B of G' in Proposition 5.



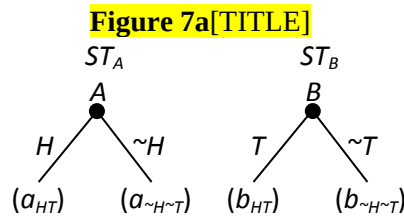
Player B will choose T over $\sim T$ if and only if b_{HT} is higher than the payoffs in $\sim T$. Again there are two cases: (a) if $a_{H\sim T} > a_{\sim H\sim T}$, B will choose T if and only if $b_{HT} > b_{H\sim T}$. Under assumption (ii) and transitivity, $b_{HT} > b_{\sim H\sim T}$. (b) if instead $a_{H\sim T} < a_{\sim H\sim T}$, B will choose T over $\sim T$ if and only if $b_{HT} > b_{\sim H\sim T}$. In both (a) and (b), $b_{HT} > b_{\sim H\sim T}$. Thus, (H, T) payoff dominates $(\sim H, \sim T)$. QED

The following reveals the robustness properties of a coordination game G with payoff dominance.

Proposition 8: Suppose (H, T) and $(\sim H, \sim T)$ are strict NE of the coordination game G. If additionally

- (i) $a_{HT} > a_{\sim H \sim T}$ and $b_{HT} > b_{\sim H \sim T}$, then (H, T) is RNE and $(\sim H, \sim T)$ is ENE
- (ii) $a_{\sim H \sim T} > a_{HT}$ and $b_{\sim H \sim T} > b_{HT}$, then $(\sim H, \sim T)$ is RNE while (H, T) is ENE.

Proof: (H, T) and $(\sim H, \sim T)$ being strict Nash Equilibrium means $a_{HT} > a_{\sim HT}$, $b_{HT} > b_{H\sim T}$ and $a_{\sim H \sim T} > a_{H\sim T}$, $b_{\sim H \sim T} > b_{\sim HT}$. This produces the reduced forms of ST_A and ST_B , respectively, given below



If (i) holds, then A chooses H in the first stage of ST_A and B's best reply to H is T by SNE of (H, T) . Thus (H, T) is SPNE in ST_A . In ST_B , B chooses T giving b_{HT} over $\sim T$ giving $b_{\sim H \sim T}$ in stage 1. A's best reply to T is H since $(\sim H, \sim T)$ is SNE, making (H, T) SPNE in ST_B and RNE. By the same token, $(\sim H, \sim T)$ cannot be the strict SPNE of any sub-tree of G' and is, thus, ENE. The proof for (ii) (the reverse of (i)) is analogous. QED

What Proposition 9 says is that payoff dominance by one of the SNEs of a coordination game G renders the game first mover invariant; the payoff dominant profile is always the unique RNE; the

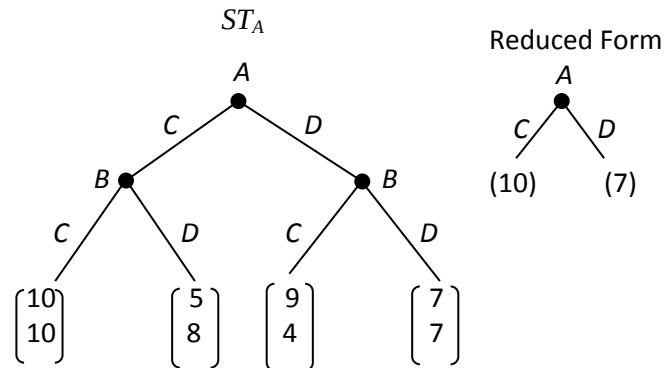
payoff dominated SNE is always ephemeral. Proposition 8 is especially useful in equilibrium selection. We consider an example of a Stag Hunt Game.

Example: Consider the Stag Hunt game G given in **Table 5** below.

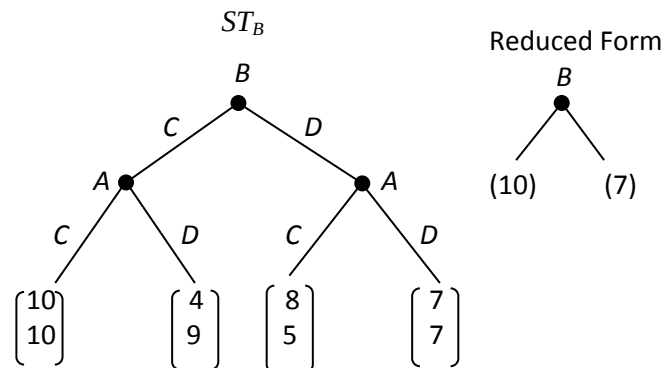
Table 5: Stag Hunt Coordination Game

		Player B	
		C	D
Player A	C	10,10	5,8
	D	9,4	7,7

The coordination game in **Table 5** has two strict Nash equilibria (C, C) and (D, D) . It is clear that (C, C) payoff dominates (D, D) . But we have $(9 - 10)(8 - 10) = 2$ for (C, C) versus $(5 - 7)(4 - 7) = 6$ for (D, D) . Thus, (D, D) risk dominates (C, C) . The two Selten-Harsanyi criteria disagree. Let us examine the robustness properties of the two SNEs. The ST_A and ST_B of G' and corresponding reduced forms are given below:



A chooses C in stage 1 while B 's best reply to C is C ($10 > 7$) and (C, C) is SPNE of ST_A . ST_B and reduced form are the following:



Thus, B starts with C in stage 1. Since $9 < 10$, C is A 's best reply to C . And (C, C) is SPNE of ST_B . Thus, (C, C) is a Robust Nash Equilibrium. (D, D) does not surface at all and is thus ephemeral.

The message of Proposition 8 is that the payoff dominated SNE of a coordination G will never be affirmed by a sequential transformation G' regardless of whether or not it exhibits risk dominance. Payoff dominance will always emerge as the solution under this sequential transformation.

V. *The Social Context and Equilibrium Selection in Coordination Games*

As observed earlier, Harsanyi and Selten (1988) argued from “collective rationality” that payoff dominance be given lexicographic priority over risk dominance. The whole community is better off with choosing the payoff dominant payoff. Risk dominance by contrast is rooted in “individual rationality.” As also observed earlier, risk dominance seems to be favored by the developments in global games and evolutionary models and Harsanyi himself switched sides in 1995. Since these models are rooted in individual rationality, that should not be surprising. And yet the problem persists (confer Cammerer, 2003).

We showed that payoff dominance’s priority may be naturally established by the sequential nature of the interaction in small groups and the imperative of backward induction rationality. Ostrom and her school (1990, 2000) documented the success of these groups in finding and sustaining payoff dominant solutions to common resource management problems. They also listed the set of circumstances (such as small groups, repeated face to face interactions, homogeneity, salience, etc.) which raise the likelihood of collective action success. Ostrom (2000, 2009) herself favored replacing rational egoism with *conditional cooperation* assumption to explain these successes. Ostrom’s approach dovetails the original Harsanyi-Selten in introducing the social context – in these small groups members behave differently than in large anonymous groups (confer especially Ostrom, 2009).

The sequential and dynamic nature of the social interactions in small long-lived groups such as studied by Ostrom and her group (1990; 2000) is undeniable. In most games it matters who makes the first move. But some games may have a payoff profile such that it does not matter who moves first. We call these games “first mover invariant games.” This is precisely the property of games with an RNE profile. We showed that the payoff dominance of an SNE is intimately connected with its robustness.

If games in small groups are played sequentially and if backward induction rationality guides agent decision-making, the payoff dominant NE profile will always be chosen regardless of who moves first. Thus, social context of the game matters. This supports the original Harsanyi-Selten position but without the need to resort to the “collective rationality” commitment of the players. Payoff dominance in coordination games render these games first mover invariant. This could be another reason why small groups in Ostrom studies are able to reach and persist in the Pareto efficient solutions. Of course one has to acknowledge that the games played in the Ostrom results may not necessarily be coordination games. We will not treat that possibility here.

VI. *Conclusion*

In this paper, we first noted that some games have too many equilibria while others have none. We deal with the static 2×2 full information game G . The “Tossing Pennies” game has no Nash equilibrium in pure strategies. Coordination games have two strict Nash equilibria in pure strategies. When the latter is the case, there is a problem of equilibrium selection. Many coordination games display the “first mover advantage” property: that is when the static game is transformed into a sequential game, who moves first has an advantage of getting his preferred payoff as SPNE of where he/she starts. In this paper, we introduce the concept of a “robust profile” of G : one which emerges as the strict SPNE of the two sub-trees resulting from the transformation of the single-stage G into a

two-stage sequential game. If G has a robust profile, then G satisfies the “no first mover advantage condition”.

We show that every strict dominant strategy equilibrium of G is robust; that not all strict Nash equilibria are robust. We show that every robust profile of G is a strict Nash equilibrium of G . We then introduce a refinement of the Nash equilibrium concept called the Robust Nash Equilibrium (RNE): a RNE is any robust profile of G . The set of RNE of G is a strict subset of the set of strict Nash equilibrium of G . In fact for the 2x2 game, the RNE of G is unique. We apply this concept to equilibrium selection problem in coordination games. Selten and Harsanyi (1988) introduced two criterion for equilibrium selection in coordination games: payoff dominance and risk dominance. If one SNE possesses both, the problem is solved. If one possesses payoff dominance but the other possesses risk dominance, there is a selection problem and Harsanyi and Selten argued for the payoff dominance from the perspective of “collective rationality.” Certain developments in global and evolutionary games seem to favor risk dominance but since collective rationality has so far played no role in these games, that is no surprise. We show that in a coordination game the payoff dominant Nash equilibrium is a robust Nash equilibrium while the payoff dominated Nash equilibrium is an Ephemeral Nash equilibrium. In other words, coordination games with a payoff dominant Nash equilibrium satisfies the “no first mover advantage condition.” A sequential transformation of the coordination game will always produce the payoff dominant profile as solution. This is achieved without a resort to collective rationality.

E. Ostrom and her school have documented the success of small groups where interactions are face-to-face and dynamic in finding and persisting in payoff superior solutions to common resource problems. Ostrom herself favored replacing “rational egoism” with “conditional cooperation” as explanation for the phenomenon. This echoes the original Harsanyi-Selten position in deviating from strict rationality. Our result shows that the cooperative outcome (the payoff dominant profile) in these groups may be being attained by the sequential and face-to-face nature of the game in these small groups. The latter can be considered the social context of the game.

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